PASSION FOR EXCELLENT

₦250

OVER 100 SOLVED PAST QUESTIONS OF MATH 106

OBL

VECTOR ANALYSIS

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THE SECRET OF MATHEMATICS

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PASSION FOR EXCELLENT

(1)

BAPTIST STUDENT FELLOWSHIP
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BSF 1: Trust in the Lord with all your heart. Never rely on what you think you know.

VECTOR ANALYSIS

This deals with physical quantities that have magnitude with or without the direction of a particular object. It is categorised into two viz:

A SCALAR QUANTITY: is a quantity that has magnitude without direction such as distance, temperature, time, Volume e.t.c. All fundamental quantities are scalar.

A VECTOR QUANTITY: Is a quantity that has magnitude with direction such as displacement, Force, Velocity e.t.c. NOT all derived quantities are Vector.

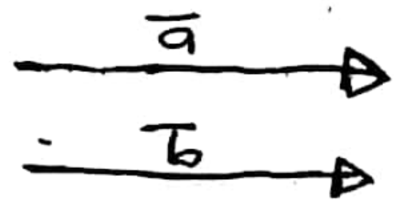
TYPES OF VECTORS

There are three major types of Vectors, namely:

1) UNIT VECTOR: Its a Vector whose magnitude is one (unity)

2) NULL VECTOR: is a vector who magnitude is zero. Its also called zero vector

3) EQUAL VECTORS: Vectors are said to be equal when they have equal magnitude in the same direction and same sense e.g two vector whose size is the same which are parallel with their arrow pointing in the same direction.



ALGEBRAIC OF VECTOR.

Find the resultant of the following vectors

A) $\vec{AB}$, $\vec{BC}$, $-\vec{DC}$

B) $\vec{AE}$, $\vec{EB}$, $\vec{BA}$

SOLUTION

A) $\vec{AB} + \vec{BC}$

$\vec{AC} + -\vec{DC}$

note $[\vec{CD} = -\vec{DC}]$

$\vec{AC} + \vec{CD}$

$\vec{AD}$ Answer

B) $\vec{AE} + \vec{EB}$

$\vec{AB} + \vec{BA} = \vec{AA} = 0$

MAGNITUDE OF A VECTOR

Given a vector

$\vec{A} = a_1 i + a_2 j + a_3 k$

magnitude of vector $\vec{A}$ =

$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Its also called modulus (mod $\vec{A}$)

UNIT VECTOR

The unit vector corresponding to given $\bar{A}$ is denoted by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{a_1\vec{i} + a_2\vec{j} + a_3\vec{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

DIRECTION COSINE OF A VECTOR OR LINE

This is the component of unit vector. Direction cosine of vector $\vec{A}$ are $\frac{a_1}{|\vec{A}|}$, $\frac{a_2}{|\vec{A}|}$, $\frac{a_3}{|\vec{A}|}$.

ANGLE BETWEEN TWO LINES OR VECTORS:

Given the following vector

$\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$

$\bar{b} = b_1 i + b_2 j + b_3 k$

If L_1, m_1 and n_1 are the corresponding direction cosine of vector $\bar{a}$ and L_2, m_2 and n_2 are that of $\bar{b}$ i.e

$$L_1 = \frac{a_1}{|\bar{a}|}, \quad m_1 = \frac{a_2}{|\bar{a}|}, \quad n_1 = \frac{a_3}{|\bar{a}|}$$

$$L_2 = \frac{b_1}{|\bar{b}|}, \quad m_2 = \frac{b_2}{|\bar{b}|}, \quad n_2 = \frac{b_3}{|\bar{b}|}$$

Then

$$L_1L_2 + m_1m_2 + n_1n_2 = \cos\theta \rightarrow \text{(I)}$$

$$L^2 + m^2 + n^2 = 1 \rightarrow \text{(II)}$$

POSITION OF VECTORS

If the position of vector A and B are $\bar{a}$ and $\bar{b}$

Given that

$\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$

$\bar{b} = b_1 i + b_2 j + b_3\bar{k}$

Then

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = (b_1 i + b_2 j + b_3 k) - (a_1 i + a_2 j + a_3 k)$

Resultant of the vector

$= \vec{OA} + \vec{OB}$

Resultant means "sum"

BSF2; Remember the Lord is everything you do and He will show you the right way

EXAMPLE

If the position vector of the points $\vec{A}$ and $\vec{B}$ are $4i+4j-7k$ and $5i-2j+16k$ respectively. Find

a) direction cosines of $\vec{AB}$

b) angle between $\vec{OA}$ and $\vec{AB}$

c) resultant of $\vec{OA}$ and $\vec{OB}$

SOLUTION

$\vec{OA} = 4i+4j-7k$

$\vec{OB} = 5i-2j+16k$

$|\vec{OA}| = \sqrt{4^2+4^2+(-7)^2} = \sqrt{16+16+49}$

$= \sqrt{81} = 9$ unit

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (5i-2j+16k) - (4i+4j-7k)$

$= i-6j+13k$

$|\vec{AB}| = \sqrt{1^2+(-6)^2+13^2} = \sqrt{1+36+169}$

$= \sqrt{1+36+169} = \sqrt{206}$ unit

direction cosine of $\vec{AB}$

$= \frac{1}{\sqrt{206}}, \frac{-6}{\sqrt{206}}$ and $\frac{13}{\sqrt{206}}$.

(b)

direction cosine of $\vec{OA}$

$= \frac{4}{9}, \frac{4}{9}$ and $-7/9$

Angle between $\vec{OA}$ and $\vec{AB}$ is given by

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

$l_1 = 1/\sqrt{206}$, $m_1 = \frac{-6}{\sqrt{206}}$, $n_1 = \frac{13}{\sqrt{20}}$

$l_2 = \frac{4}{9}$, $m_2 = \frac{4}{9}$ $n_2 = -7/$

$\cos\theta = \frac{4}{9}\left(\frac{1}{\sqrt{206}}\right) + \frac{4(-6)}{9\sqrt{206}} + \frac{-7(1}{9\sqrt{2}}$

$\cos\theta = \frac{4}{9\sqrt{206}} - \frac{24}{9\sqrt{206}} - \frac{91}{9\sqrt{206}}$

$\cos\theta = \frac{-111}{9\sqrt{206}}$

$\theta = \cos^{-1}\left(\frac{-37}{3\sqrt{206}}\right)$

PERPENDICULARITY OF VECTORS

Two vectors are said to be perpendicular to each other when they are at right angle to each other i.e angle between vector $\vec{A}$ and $\vec{B}$ is $90°$

That is, $\cos\theta = \cos 90 =$

$\boxed{l_1l_2 + m_1m_2 + n_2n_2 = 0}$

EXAMPLE

Show that $\bar{a} = 9i + j - 6\bar{k}$ and $\bar{b} = 4i - 6j + 5k$ are at right angle

SOLUTION

$\bar{a} = 9i + j - 6\bar{k}$

$|\bar{a}| = \sqrt{9^2 + 1^2 + (-6)^2} = \sqrt{81 + 36 + 1}$

$= \sqrt{118}$

$\bar{b} = 4i - 6j + 5k$

$|\bar{b}| = \sqrt{4^2 + (-6)^2 + 5^2} = \sqrt{16 + 36 + 25}$

$|\bar{b}| = \sqrt{77}$

$l_1 = \frac{9}{\sqrt{118}}$, $m_1 = \frac{1}{\sqrt{118}}$, $n_1 = \frac{-6}{\sqrt{118}}$

$l_2 = \frac{4}{\sqrt{77}}$, $m_2 = \frac{-6}{\sqrt{77}}$, $n_1 = \frac{5}{77}$

$\cos\theta = \frac{9 \times 4}{\sqrt{118} \times \sqrt{77}} - \frac{6 \times 1}{\sqrt{118} \times \sqrt{77}} - \frac{6 \times 5}{\sqrt{118} \times \sqrt{77}}$

$\cos\theta = \frac{36 - 6 - 30}{\sqrt{118} \times \sqrt{77}} = 0$

$\theta = \cos^{-1} 0 = \underline{\underline{90^\circ}}$

EXAMPLE

Prove that three points whose position vectors are $i + 2j + 3k$, $-i - j + 8k$ and $-4i + 4j + 6k$ form and equilateral triangle

SOLUTION

$\vec{OA} = i + 2j + 3k$

$\vec{OB} = -i - j + 8k$

$\vec{OC} = -4i + 4j + 6k$

$\vec{AB} = \vec{OB} - \vec{OA}$

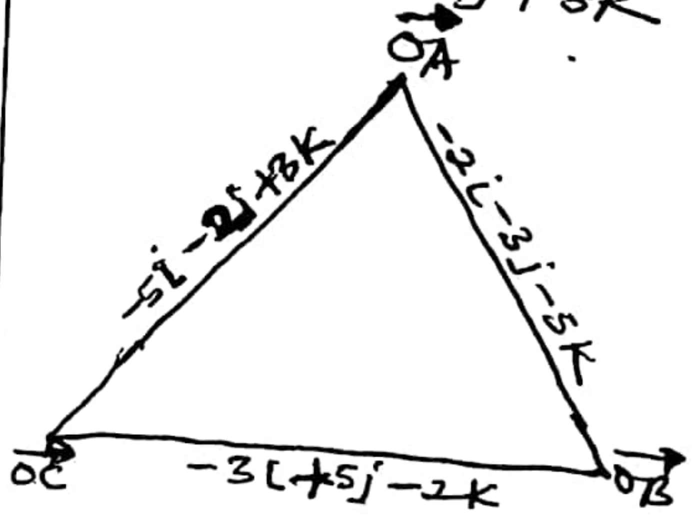
$= -2i - 3j - 5k$

$\vec{BC} = \vec{OC} - \vec{OB}$

$-3i + 5j - 2k$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= -5i - 2j + 3k$



$|\vec{AB}| = \sqrt{(-2)^2 + (-3)^2 + (-5)^2} = \sqrt{38}$

$|\vec{BC}| = \sqrt{(-3)^2 + 5^2 + (-2)^2} = \sqrt{38}$

$|\vec{AC}| = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38}$

All sides are equal

BSF3: You can be ruined by the talk of godless people, but wisdom of the righteous can save you.

EXAMPLE

Find the direction cosine of

a) $(-9, 0, 0)$ and $(1, 1, 1)$

b) $(1, 1, -1)$ and $(0, 1, -1)$

SOLUTION

(a)

$\vec{OA} = -9i + 0j + 0k$

$\vec{OB} = i + j + k$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (1+9)i + j + k$

$|\vec{AB}| = \sqrt{(1+9)^2 + 1^2 + 1^2}$

$= \sqrt{(1+9)^2 + 2}$

$d.c = \left(\frac{1+9}{\sqrt{(1+9)^2+2}}, \frac{1}{\sqrt{(1+9)^2+2}}, \frac{1}{\sqrt{(1+9)^2+2}} \right)$

EXERCISE

Attempt (b)

SOLVED PAST QUESTION

2010 NO22: Given that $a = 3i - 2j + k$, $b = 2i - 4j - 3k$, $c = -i + 2j + 2k$. Then the magnitude of $(a+b+c)$ is

A) none B) $\sqrt{15} + \sqrt{27} + 3$ C) $4\sqrt{2}$ D) $16\sqrt{2}$ E) 7

SOLUTION

$a = 3i - 2j + k$

$b = 2i - 4j - 3k$

$c = -i + 2j + 2k$

$\bar{a} + \bar{b} + \bar{c} = 4i - 4j + 0k$

$|a+b+c| = \sqrt{4^2 + (-4)^2 + 0^2} = \sqrt{16+16}$

$= 4\sqrt{2}$

Answer is C

NO25: For what value of a are the vectors $a = 2i + aj + k$ and $b = 4i - 2j - 2k$ are perpendicular A) 3 B) -3 C) 0 D) 4 E) none

SOLUTION

$a = 2i + aj + k$

$a \cdot b = 0$ or

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$a \cdot b = (2i + aj + k)(4i - 2j - 2k) = 0$

$= 8 - 2a - 2 = 0$

$2a = 6$

$a = 3$

Answer is A

2009 No 9: which of the following gives the expression for the cosine of the angle between the vectors
$\bar{a} = 2i + 2j - \bar{k}$ and $\bar{b} = 6i - 3j + 2\bar{k}$

A) $\frac{6}{21}$ B) $\frac{4}{\sqrt{21}}$ C) $\frac{4}{21}$ D) $\frac{\sqrt{4}}{21}$ E) $\frac{8}{21}$

SOLUTION

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

OR

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| \cdot |\bar{b}|}$$

$$\cos\theta = \frac{(2i + 2j - k) \cdot (6i - 3j + 2k)}{\sqrt{2^2 + 2^2 + (-1)^2} \cdot \sqrt{6^2 + (-3)^2 + 2^2}}$$

$$\cos\theta = \frac{12 - 6 - 2}{\sqrt{9}\,\sqrt{49}} = \frac{4}{3 \times 7} = \frac{4}{21}$$

Answer is C

COLLINEARITY OF VECTORS

Vectors are said to be collinear if sum of their coefficient is zero e.g

If $x\vec{a} + y\vec{b} + z\vec{c} = 0$

and $x + y + z = 0$

Then $\bar{a}$, $\bar{b}$ and $\bar{c}$ are collinear

EXAMPLE

Show that if $7\vec{AB} - 2\vec{AC}$ $5\vec{AD} = 0$, the point B, C are collinear

SOLUTION

$7 - 2 - 5 = 0$

Hence they are collinear

MULTIPLE OF SCALAR

Three vectors are said to be collinear if their relative position are multiple of scalar e.g integral multiple of number

EXAMPLE

Show that the following point are collinear $-2\vec{a} + 3\vec{b} + 5\vec{c}$, $\vec{a} + 2\vec{b} + 3\vec{c}$, $7\vec{a} - \vec{c}$.

SOLUTION

let $\vec{A} = -2\vec{a} + 3\vec{b} + 5\vec{c}$

$B = \vec{a} + 2\vec{b} + 3\vec{c}$

$C = 7\vec{a} - \vec{c}$

$\vec{AC} = \lambda \vec{AB}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= 9\vec{a} - 3\vec{b} - 6\vec{c}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= 3\vec{a} - \vec{b} - 2\vec{c}$

BSF 4: Sensible people keep quiet about what they know, but stupid people advertise their ignorance

$\vec{AC} = \lambda \vec{AB}$

$9\vec{a} - 3\vec{b} - 6\vec{c} = \lambda(-3\vec{a} - \vec{b} - 2\vec{c})$

$9\vec{a} - 3\vec{b} - 4\vec{c} = -3\lambda\vec{a} - \lambda\vec{b} - 2\lambda\vec{c}$

Equating the corresponding terms

$3\lambda\vec{a} = 9\vec{a} \longrightarrow (i)$

$\lambda = \frac{9\vec{a}}{3\vec{a}} = 3$

OR

$-\lambda\vec{b} = -3\vec{b} \longrightarrow (ii)$

$\lambda = 2$

OR

$-2\lambda\vec{c} = -6\vec{c} \longrightarrow (iii)$

$\lambda = 3$

Thus

$\vec{AC} = 3\vec{AB}$

$\vec{A}, \vec{B}, \vec{C}$ are collinear

EXAMPLE

Find the value of x if vectors

$\vec{A} =$ [illegible] $i + 3j + xk$

$\vec{B} = 4i + 2j + 3k$

$\vec{C} = 7i + 0j - k$

are collinear.

SOLUTION

$\vec{AC} = \lambda AB$

$\vec{AC} = \vec{OC} - \vec{OA}$

$\vec{AC} = 6i - 3j - (1+x)k$

$\vec{AB} = 3i -$ [illegible]$j + (3-x)k$

$\vec{AB} = \vec{OB} - \vec{OA}$

$6i - 3j - (1+x)k = \lambda(3i -$ [illegible]$j + (3-$

$3\lambda i = 6i \longrightarrow (i)$

OR

$-\lambda i = -3j \longrightarrow (ii)$

$\lambda = 2$

$-(1+x)\cancel{k} = (3-x)\lambda \cancel{k}$

$-(1+x) = (3-x)\times 2$

$-(1+x) = 6 - 2x$

$-1 - x = 6 - 2x$

$-x + 2x = 6 + 1$

$x = \underline{7}$

COPLANARITY OF VECTOR

Coplanar vector are those vectors which are parallel to the same plane. Any vector $\vec{r}$ is coplanar with given non-collinear as a linear combination of given vectors 1-

$\vec{r} = x\vec{a} + y\vec{b}$ where c and y are scalar

EXAMPLE

xamine whether the following ector are Coplanar or not

$\vec{a} + 2\vec{b} + \vec{c}$, $\vec{a} + 19\vec{b} - 7\vec{c}$

d $2\vec{a} + 3\vec{b} - 4\vec{c}$

SOLUTION

$= -\vec{a} + 2\vec{b} + \vec{c}$

$= \vec{a} + 19\vec{b} - 7\vec{c}$

$= 2\vec{a} + 3\vec{b} - 4\vec{c}$

$\vec{c} = x\vec{a} + y\vec{b}$

where x and y are scalar

$a + 3\bar{b} - 4c = x(-\bar{a} + 2\bar{b} + \bar{c})$
$+ y(\bar{a} + 19\bar{b} - 7\bar{c})$

$+ 3\bar{b} - 4\bar{c} = (-x+y)\bar{a} +$
$(2x+19y)\bar{b} +$
$(x-7y)\bar{c}$

mparing the coefficient $\bar{a}, \bar{b}, \bar{c}$ on both les we have

$-x+y = 2 \rightarrow$ (I)

$2x+19y = 3 \rightarrow$ (II)

$x - 7y = 4 \rightarrow$ (III)

olve two equation o find x and y and substitute them in third equation

If L.H.S = R.H.S

Then, they are Coplanar

Consider eq (I) and (III)

$-x + y = 2$ — ~~(I)~~

$x - 7y = -4$ → ~~(III)~~

add up

$-6y = -2$

$y = 2/6 = 1/3$

$-x + y = 2$

$-x + y = 2$

$x = y - 2$

$x = \frac{1}{3} - \frac{2}{1} = \frac{1-6}{3} = \frac{-5}{3}$

Substitute x and y in eq (II)

$2x + 19y = 3$

$2(-5/3) + 19(1/3) = 3$

$\frac{-10+19}{3} = 3$

$\frac{9}{3} = 3$

$3 = 3$

L.H.S = R.H.S

Hence they are Collinear.

BSF5: It is the Lord who gives wisdom; from him come knowledge and understanding.

OPERATION SHARP-SHARP

Three vectors are coplanar when their determinant equal Zero

From the previous example,

$\bar{a} = -\vec{a} + 2b + c$

$\bar{b} = a + 19b - 7c$

$\bar{c} = 2a + 3b - 4c$

$$\begin{array}{ccc} a & b & c \\ + & - & + \end{array}$$

$$\begin{vmatrix} -1 & 2 & 1 \\ 1 & 19 & -7 \\ 2 & 3 & -4 \end{vmatrix}$$

$$+(-1)\begin{vmatrix} 19 & -7 \\ 3 & -4 \end{vmatrix} - 2\begin{vmatrix} 1 & -7 \\ 2 & -4 \end{vmatrix} + 1\begin{vmatrix} 1 & 19 \\ 2 & 3 \end{vmatrix}$$

$$= -1((19\times -4) - (-7\times 3)) - 2((1\times -4) - (2\times -7)) + 1((1\times 3) - (2\times 19))$$

$$= -1(-76+21) - 2(-4+14) + 1(3-38)$$

$$= -1(-55) - 2(10) + 1(-35)$$

$$55 - 20 - 35 = 0$$

EXAMPLE

If $5i + mj - 3k$, $2i - 3j + 5k$ and $i + j - 6k$ are coplanar Find m

SOLUTION

Using determinant method

$$\begin{array}{ccc} i & j & k \\ + & - & + \end{array}$$

$$\begin{vmatrix} 5 & m & -3 \\ 2 & -3 & 5 \\ 1 & 1 & -6 \end{vmatrix} = 0$$

$$+5\begin{vmatrix} -3 & 5 \\ 1 & -6 \end{vmatrix} - m\begin{vmatrix} 2 & 5 \\ 1 & -6 \end{vmatrix} - 3\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$$

$$5(18-5) - m(-12-5) - 3(2+3) =$$

$$5(13) - m(-17) - 3(5) = 0$$

$$65 + 17m - 15 = 0$$

$$50 + 17m = 0$$

$$17m = -50$$

$$m = -50/17$$

LINEARLY DEPENDENT VECTOR

Vector $\vec{a_1}\ \vec{a_2}\ \vec{a_3} --- \vec{a_n}$ are said to be linearly dependent if

$k_1\vec{a_1} + k_2\vec{a_2} + k_3\vec{a_3} + k_n\vec{a_n} = 0$

For at least one k different from zero i.e k_1 k_2 may be zero but k_3 will be real number provided we have three given vectors or two k's or all may not be zero

not if

$k_1 = k_2 = k_3 = -- k_n = 0$

then they are linearly dependent

EXAMPLE

Show that the vector $i+j+2k$, $i+2j+5k$ and $5i+3j+4k$ are linearly dependent or otherwise

SOLUTION

$k_1(i+j+2k) + k_2(i+2j+5k) + k_3(5i+3j+4k) = 0$ 12

equating the coefficient of i, j and k to zero

$k_1 + k_2 + 5k_3 = 0 \rightarrow (1)$

$k_1 + 2k_2 + 3k_3 = 0 \rightarrow (2)$

$2k_1 + 5k_2 + 4k_3 = 0 \rightarrow (3)$

from eq (2) and (3)

$k_1 + 2k_2 + 3k_3 = 0 \quad \times 4$

$2k_1 + 5k_2 + 4k_3 = 0 \quad \times 3$

$4k_1 + 8k_2 + 12k_3 = 0$

$6k_1 + 15k_2 + 12k_3 = 0$

subtract

$2k_1 + 7k_2 = 0 \rightarrow (4)$

from eq (1) and (2)

$k_1 + k_2 + 5k_3 = 0 \quad \times 3$

$k_1 + 2k_2 + 3k_3 = 0 \quad \times 5$

$3k_1 + 3k_2 + 15k_3 = 0$

$5k_1 + 10k_2 + 15k_3 = 0$

subtract

$2k_1 + 7k_2 = 0 \rightarrow (5)$

$2k_1 + 7k_2 = 0 \rightarrow (4)$

$k_1 = 0 \quad k_2 = 0$

Lit 6: Reverence for the Lord is an education in itself. You must be humble before you can ever receive honours.

from eq (2) and (3)

$k_1 + 2k_2 + 3k_3 = 0 \quad \times 2$

$2k_1 + 5k_2 + 4k_3 = 0 \quad \times 1$

$2k_1 + 4k_2 + 6k_3 = 0$

$2k_1 + 5k_2 + 4k_3 = 0$

Subtract

$-k_2 + 2k_3 = 0$

$k_2 = 2k_3 \longrightarrow \boxtimes$

Substitute for k_2 in eq (1)

$k_1 + k_2 + 5k_3 = 0$

$k_1 + 2k_3 + 5k_3 = 0$

$k_1 = -7k_3 \longrightarrow \boxtimes$

if $k_3 = 1$

$k_2 = 2 \quad k_1 = -7$

Then in eq (2)

$k_1 + 2k_2 + 3k_3 = 0$

$-7 + 2(2) + 3(1) = 0$

$-7 + 4 + 3 = 0$

Thus the given vectors are linearly dependent

note: It will also satisfy equation (1) and (3)

SHORT - CUT

Three vectors are linearly dependent whe their determinant equal zero

$$\begin{array}{ccc} i & j & k \\ + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 3 \\ 2 & 5 & 4 \end{vmatrix}$$

$+1[-7] - 1[-2] + 5[1] = 0$

CHOP → BISCUITS

a) show that the vectors

$\vec{a} = 2i - 2j + k$

$\vec{b} = i + 4j - k$

$\vec{c} = 4i + 6j - 3k$ are linearly Independent

HINT: determinant $\neq 0$

b) prove that (1, 3, 2) (1, −7, −8). and (1, 1, −1) are linear Independent

c) prove that (2, −2, 1) (1, 4, −1) and (5, 3, 4) are linearly dependent

SOLVED PAST QUESTION

2009 No 7: If Vectors $\bar{i}+\bar{j}+2\bar{k}$, $\bar{i}+p\bar{j}+5\bar{k}$ and $5\bar{i}+3\bar{j}+4\bar{k}$ are linearly dependent, the value p is

(A) 2 (B) 4 (C) 3 (D) -3 (E) -2

SOLUTION

$$\begin{array}{ccc} i & j & k \\ + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & p & 5 \\ 5 & 3 & 4 \end{vmatrix} = 0$$

$$+1\begin{bmatrix} p & 5 \\ 3 & 4 \end{bmatrix} -1\begin{bmatrix} 1 & 5 \\ 5 & 4 \end{bmatrix} +2\begin{bmatrix} 1 & p \\ 5 & 3 \end{bmatrix} = 0$$

$$+1[4p-15]-1[4-25]+2[3-5p]=0$$

$$4p-15+21+6-10p=0$$

$$-6p=-27+15$$

$$-6p=-12$$

$$p=2$$

Answer is A

DOT OR SCALAR PRODUCT

The scalar or dot product of two vector $\bar{a}$ and $\bar{b}$ with an angle θ between them as $ab\cos\theta$. It is denoted by $\bar{a}\cdot\bar{b}$

Hence $\bar{a}\cdot\bar{b} = ab\cos\theta$

If the two vector are perpendicular then

$$\bar{a}\cdot\bar{b}=0$$

$$\bar{a}\cdot\bar{b} = ab\cos\frac{\pi}{2} = 0$$

Let $ab\cos\theta = 0$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}0$$

$$\theta = \frac{\pi}{2}$$

$$\bar{a} \perp \bar{b}$$

APPLICATION OF DOT PRODUCT

A) PROJECTION OF VECTORS

Projection of vector $\bar{b}$ on $\bar{a}$ $= |b|\cos\theta$

$$|a||b|\cos\theta = \bar{a}\cdot\bar{b}$$

$$|b|\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|} = \bar{b}\cdot\hat{a}$$

Vector projection of $\bar{b}$ on $\bar{a}$

$$= \frac{\bar{a}\cdot\bar{b}}{a}\left(\frac{\bar{a}}{a}\right)$$

$$= \bar{a}\frac{(\bar{a}\cdot\bar{b})}{a^2} = (\bar{b}\cdot\hat{a})\hat{a}$$

Vector projection of $\bar{a}$ on $\bar{b}$

$$= \frac{(\bar{a}\cdot\bar{b})\bar{b}}{b^2} = (\bar{a}\cdot\hat{b})\hat{b}$$

BSF7: Pay attention to what you are taught, and you will be successful; trust in the Lord and you will be happy.

EXAMPLE

Find the projection of the vector $\bar{a} = i - 2j + k$ on the vector $\bar{b} = 4i - 4j + 7k$

SOLUTION

projection of $\bar{a}$ on $\bar{b}$ = $\bar{a} \cdot \hat{b}$ where

$\hat{b}$ = Unit vector of $\bar{b}$

$$\hat{b} = \frac{\bar{b}}{|\bar{b}|} = \frac{4i - 4j + 7k}{\sqrt{4^2 + (4)^2 + 7^2}}$$

$$\hat{b} = \frac{4i - 4j + 7k}{\sqrt{16 + 16 + 49}}$$

$$\hat{b} = \frac{4i - 4j + 7k}{9}$$

$$\bar{a} \cdot \hat{b} = (i - 2j + k) \cdot \left(\frac{4i - 4j + 7k}{9}\right)$$

$$= \frac{4 + 8 + 7}{9} = \frac{19}{9}$$

Vector projection of $\bar{a}$ on $\bar{b}$ = $(\bar{a} \cdot \hat{b})\bar{b}$

$$= \frac{19}{9}(4i - 4j + 7k)$$

EXAMPLE

Determine the value of m so that $\bar{a} = 2i + mj + k$ and $\bar{b} = 4i - 2j - 2k$ are perpendicular

SOLUTION

$\bar{a} \cdot \bar{b} = 0$ $\quad \bar{a} \perp \bar{b}$

$(2i + mj + k) \cdot (4i - 2j - 2k) = 0$

$8 - 2m - 2 = 0$

$2m = 6 \quad m = 2$

CHOP → CHIN CHIN

a) Show that $\bar{a} = 3i - 2j$, $\bar{b} = i - 3j + 5k$, $c = 2i + j$ formed a right angle triangle

HINT: $|AB|^2 + |BC|^2 = |AC|$

or when $\bar{a} = \bar{b} + \bar{c}$ or $\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c} = 0$

b) Find the Cosine of the angle between $\bar{a} = 2i + 2j - k$ and $\bar{b} = 6i - 3j + 2k$

HINT $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$

ITS TIME TO WAKE UP!!!

SOLVED PAST QUESTION

2008/2009 NO 1 find the Component of the vector $4i - 4j + 7k$ on the vector $i + j + k$ A) $11/\sqrt{3}$ B) $7/\sqrt{3}$ C) $3/\sqrt{3}$ D) $7/3$ E) $11/3$

SOLUTION

Let $\bar{a} = 4i - 4j + 7k$

$\bar{b} = i + j + k$

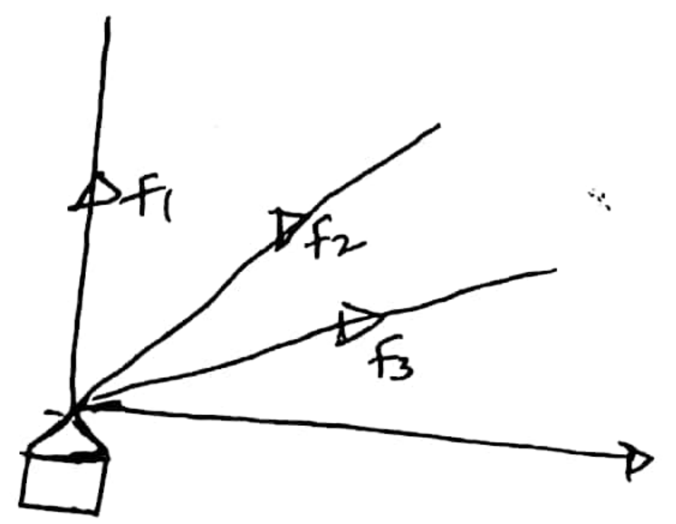
$$\bar{a} \cdot \hat{b} = (4i - 4j + 7k) \cdot \frac{(i + j + k)}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$= \frac{4 - 4 + 7}{\sqrt{3}} = 7/\sqrt{3}$$

Answer is B

B) RESULTANT FORCE AND WORDONE

Given three forces acting on a object



Resultant force $= F_1 + F_2 + F_3$

$F = \sum F_n$

$F_n =$ magnitude of force $\times$ Unit Vector

$n = 1, 2, 3 \ldots$

Wordone $= \bar{F} \bullet \bar{d}$ ← scalar product

$\bar{d} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

EXAMPLE

Forces of magnitude 5, 3, 1 unit act in the direction $6i + 2j + 3k$, $3i - 2j + 6k$ and $2i - 3j - 6k$ respectively on a particle which is displaced from the point $(2, -1, -3)$ to $(5, -1, 1)$ find the workdone by the force.

SOLUTION

$F_n =$ magnitude of force $\times$ Unit vector

$$F_1 = 5 \text{ times } \frac{(6i + 2j + 3k)}{\sqrt{6^2 + 2^2 + 3^2}}$$

$$F_2 = 3 \text{ times } \frac{(3i - 2j + 6k)}{\sqrt{3^2 + 2^2 + 6^2}}$$

$$F_3 = 1 \text{ times } \frac{(2i - 3j - 6k)}{\sqrt{2^2 + 3^2 + 6^2}}$$

$F = F_1 + F_2 + F_3$

$$F = \frac{5(6i + 2j + 3k) + 3(3i - 2j + 6k) + 1(2i - 3j - 6k)}{\sqrt{49}}$$

BSF 8: Better to eat a dry crust of bread with peace of mind than to have a banquet in a house full of trouble.

$$F = \frac{\cancel{41}i \cancel{+} j + 27k}{7}$$

Workdone $= \vec{F} \cdot \vec{d}$

$\vec{d} = \vec{OB} - \vec{OA} = \vec{AB}$

$\vec{OA} = 2i - j - 3k$

$\vec{OB} = 5i - j + k$

$\vec{d} = 3i + 0j + 4k$

Workdone $= \left(\frac{41i + j + 27k}{7}\right) \cdot (3i + 4k)$

$= \frac{41 + 108}{7} = \frac{231}{7}$

$= \underline{33 \text{ Unit}}$

EXAMPLE

A force of magnitude 6 units acting parallel to $2i - 2j + k$ displaces the point of application from $(1,2,3)$ to $(5,4,7)$. Find the workdone by the forces.

SOLUTION

$\vec{F}_n = 2i - 2j + k$

$\vec{A} = i + 2j + 3k \quad (1, 2, 3)$

$\vec{B} = 5i + 4j + 7k \quad (5, 4, 7)$

$\vec{d} = \vec{OB} - \vec{OA}$

$\vec{d} = 4i + 2j + 4k$

$$F = 6 \times \frac{2i - 2j + k}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

$$\vec{F} = \cancel{6} \text{ times } \left(\frac{2i - 2j + k}{\cancel{3}}\right)$$

$\vec{F} = 2(2i - 2j + k)$

$= 4i - 4j + 2k$

Workdone $= \vec{F} \cdot \vec{d}$

$= (4i - 4j + 2k) \cdot (4i + 2j + 4k)$

$= 16 + 8 - 8$

$= 16$ Unit

CHOP - GROUNDNUT

a) A particle acted on by two forces $4i + j - 3k$ and $3i + j - k$ is displaced from the point $i + 2j + k$ to the point $5i + 4j + k$. Find the workdone by the forces.

b) Force of magnitude 3 and 2 in the direction $i - 2j + 2k$ and $2i - 3j - 6k$ respectively act on a particle which is displaced from the point $(2, -1, -3)$ to $(5, -1, 1)$. Find the workdone by the Force

SOLVED PAST QUESTION 2009 No 7: If Vectors $\bar{i}+\bar{j}+2\bar{k}$, $\bar{i}+p\bar{j}+5\bar{k}$ and $5\bar{i}+3\bar{j}+4\bar{k}$ are linearly dependent, the value p is (A) 2 (B) 4 (C) 3 (D) −3 (E) −2

SOLUTION

$$\begin{array}{ccc} i & j & k \\ + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & p & 5 \\ 5 & 3 & 4 \end{vmatrix} = 0$$

$$+1\begin{bmatrix} p & 5 \\ 3 & 4 \end{bmatrix} - 1\begin{bmatrix} 1 & 5 \\ 5 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & p \\ 5 & 3 \end{bmatrix} = 0$$

$$+1[4p-15] - 1[4-25] + 2[3-5p] = 0$$

$$4p - 15 + 21 + 6 - 10p = 0$$

$$-6p = -27 + 15$$

$$-6p = -12$$

$$p = 2$$

Answer is A

DOT OR SCALAR PRODUCT

The scalar or dot product of two vector $\bar{a}$ and $\bar{b}$ with an angle θ between them as $ab\cos\theta$. It is denoted by $\bar{a}\cdot\bar{b}$

Hence $\bar{a}\cdot\bar{b} = ab\cos\theta$

If the two vector are perpendicular then

$$\bar{a}\cdot\bar{b} = 0$$

$$\bar{a}\cdot\bar{b} = ab\cos\frac{\pi}{2} = 0$$

Let $ab\cos\theta = 0$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}0$$

$$\theta = \frac{\pi}{2}$$

$$\bar{a} \perp \bar{b}$$

APPLICATION OF DOT PRODUCT

A) PROJECTION OF VECTORS

Projection of vector $\bar{b}$ on $\bar{a}$ $= |b|\cos\theta$

$$|a||b|\cos\theta = \bar{a}\cdot\bar{b}$$

$$|b|\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|} = \bar{b}\cdot\hat{a}$$

Vector projection of $\bar{b}$ on $\bar{a}$

$$= \frac{\bar{a}\cdot\bar{b}}{a}\left(\frac{\bar{a}}{a}\right)$$

$$= \bar{a}\frac{(\bar{a}\cdot\bar{b})}{a^2} = (\bar{b}\cdot\hat{a})\hat{a}$$

Vector projection of $\bar{a}$ on $\bar{b}$

$$= \frac{(\bar{a}\cdot\bar{b})\bar{b}}{b^2} = (\bar{a}\cdot\hat{b})\hat{b}$$

BSF'7: Pay attention to what you are taught, and you will be successful; trust in the Lord and you will be happy.

EXAMPLE

Find the projection of the vector $\bar{a} = i - 2j + k$ on the vector $\bar{b} = 4i - 4j + 7k$

SOLUTION

projection of $\bar{a}$ on $\bar{b}$ =

$\bar{a} \cdot \hat{b}$ where

$\hat{b}$ = Unit vector of $\bar{b}$

$$\hat{b} = \frac{\bar{b}}{|\bar{b}|} = \frac{4i - 4j + 7k}{\sqrt{4^2 + (4)^2 + 7^2}}$$

$$\hat{b} = \frac{4i - 4j + 7k}{\sqrt{16 + 16 + 49}}$$

$$\hat{b} = \frac{4i - 4j + 7k}{9}$$

$$\bar{a} \cdot \hat{b} = (i - 2j + k) \cdot \left(\frac{4i - 4j + 7k}{9}\right)$$

$$= \frac{4 + 8 + 7}{9} = \frac{19}{9}$$

Vector projection of $\bar{a}$ on $\bar{b}$ = $(\bar{a} \cdot \hat{b}) \hat{b}$

$$= \frac{19}{9}(4i - 4j + 7k)$$

EXAMPLE

Determine the value of m so that $\bar{a} = 2i + mj + k$ and $\bar{b} = 4i - 2j - 2k$ are perpendicular

SOLUTION

$\bar{a} \cdot \bar{b} = 0$ $\quad \bar{a} \perp \bar{b}$

$(2i + mj + k) \cdot (4i - 2j - 2k) = 0$

$8 - 2m - 2 = 0$

$2m = 6 \quad m = 2$

CHOP → CHIN CHIN

a) Show that $\bar{a} = 3i - 2j -$ $\bar{b} = i - 3j + 5k$ $c = 2i + j$ formed a right angle triangle

HINT : $|AB|^2 + |BC|^2 = |AC|$
or when $\bar{a} = \bar{b} + \bar{c}$ (
$\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c} = 0$

b) Find the cosine of the angle between $\bar{a} = 2i + 2j - k$ and $\bar{b} = 6i - 3j + 2k$

HINT $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$

ITS TIME TO WAKE UP!!!

SOLVED PAST QUESTION

2008/2009 No 1 find the component of the vector $4i-4j+7k$ on the vector $i+j+k$ A) $11/\sqrt{3}$ B) $7/\sqrt{3}$ C) $3/\sqrt{3}$ D) $7/3$ E) $11/3$

SOLUTION

Let $\bar{a} = 4i-4j+7k$

$\bar{b} = i+j+k$

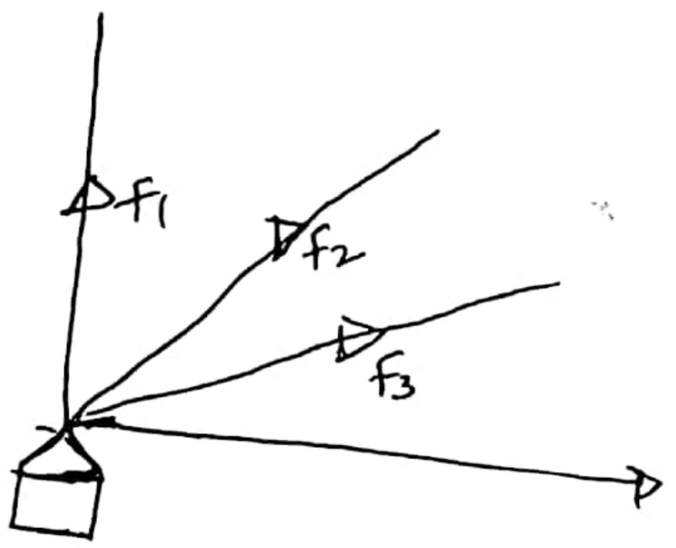
$$\bar{a}\cdot\hat{b} = (4i-4j+7k)\cdot\frac{(i+j+k)}{\sqrt{1^2+1^2+1^2}}$$

$$= \frac{4-4+7}{\sqrt{3}} = 7/\sqrt{3}$$

Answer is B

B) RESULTANT FORCE AND WORKDONE

Given three forces acting on a object



Resultant force $F = \sum F_n = F_1+F_2+F_3$

F_n = magnitude of force × unit vector

$n = 1, 2, 3 \ldots$

Workdone $= \bar{F}\bullet\bar{d}$ scalar product

$\bar{d} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

EXAMPLE

Forces of magnitude 5, 3, 1 unit act in the direction $6i+2j+3k$, $3i-2j+6k$ and $2i-3j-6k$ respectively on a particle which is displaced from the point $(2,-1,-3)$ to $(5,-1,1)$ find the workdone by the force.

SOLUTION

F_n = magnitude of force × unit vector

$$F_1 = 5 \text{ times } \frac{(6i+2j+3k)}{\sqrt{6^2+2^2+3^2}}$$

$$F_2 = 3 \text{ times } \frac{(3i-2j+6k)}{\sqrt{3^2+2^2+6^2}}$$

$$F_3 = 1 \text{ times } \frac{(2i-3j-6k)}{\sqrt{2^2+3^2+6^2}}$$

$F = F_1+F_2+F_3$

$$F = \frac{5(6i+2j+3k)+3(3i-2j+6k)+1(2i-3j-6k)}{\sqrt{49}}$$

BSF 8: Better to eat a dry crust of bread with peace of mind than to have a banquet in a house full of trouble.

$F = \frac{41i + j + 27k}{7}$

Workdone $= F \cdot d$

$\vec{d} = \vec{OB} - \vec{OA} = \vec{AB}$

$\vec{OA} = 2i - j - 3k$

$\vec{OB} = 5i - j + k$

$\vec{d} = 3i + 0j + 4k$

Workdone $= \left(\frac{41i + j + 27k}{7}\right) \cdot (3i + 4k)$

$= \frac{41 + 108}{7} = \frac{231}{7}$

$= \underline{33 \text{ Unit}}$

EXAMPLE

A force of magnitude 6 units acting parallel to $2i - 2j + k$ displaces the point of application from $(1, 2, 3)$ to $(5, 4, 7)$. Find the workdone by the forces.

SOLUTION

$\vec{F_n} = 2i - 2j + k$

$\vec{A} = i + 2j + 3k \quad (1, 2, 3)$

$\vec{B} = 5i + 4j + 7k \quad (5, 4, 7)$

$\vec{d} = \vec{OB} - \vec{OA}$

$\vec{d} = 4i + 2j + 4k$

$F = 6 \times \frac{2i - 2j + k}{\sqrt{2^2 + (-2)^2 + 1^2}}$

$\vec{F} = 6 \text{ times } \left(\frac{2i - 2j + k}{3}\right)$

$\vec{F} = 2(2i - 2j + k)$

$= 4i - 4j + 2k$

Workdone $= \vec{F} \cdot \vec{d}$

$= (4i - 4j + 2k) \cdot (4i + 2j + 4k)$

$= 16 + 8 - 8$

$= 16 \text{ Unit}$

CHOP - GROUNDNUT

a) A particle acted on by two forces $4i + j - 3k$ and $3i + j - k$ is displaced from the point $i + 2j + k$ to the point $5i + 4j + k$. Find the workdone by the forces.

b) Force of magnitude 3 and 2 in the direction $i - 2j + 2k$ and $2i - 3j - 6k$ respectively act on a particle which is displaced from the point $(2, -1, -3)$ to $(5, -1, 1)$. Find the workdone by the force

SOLVED PAST QUESTION

2009: No 21: A particle is displaced from the point $i+2j$ to the point $5i+4j$ by constant forces $4i+3j$, $3i+2j$. Find the workdone.
A) 38 B) 37 C) 39 D) 28 E) 27

SOLUTION

$F = F_1 + F_2 = (4i+3j)+(3i+2j)$

$F = 7i+5j$

$d = \vec{OB} - \vec{OA} = (5i+4j)-(i+2j)$

$= 4i+2j$

Workdone $= \vec{F}\cdot\vec{d} = (7i+5j)\cdot(4i+2j)$

$= 28 + 10 = 38$

Answer is A

2008/2009 No 21: Find the value of n such that the vector $3i+9j+2k$ is perpendicular to the vector $5i-nj-3k$ A) -1 B) 1 C) 9 D) 4 E) 2

SOLUTION

$\bar{a} = 3\bar{i}+9\bar{j}+2\bar{k}$

$\bar{b} = 5\bar{i}-n\bar{j}-3\bar{k}$

$\bar{a}\cdot\bar{b} = 0$

$(3i+9j+2k)\cdot(5i-nj-3k) = 0$

$15 - 9n - 6 = 0$

$n = 1$

Answer is (B)

No 7: If vectors $2i+j-$ $i+nk$ and $4i+3j-k$ are linearly dependent, calculate the value of n A) -4 B) 3 C) 4 D) 2 E) -2.

SOLUTION

$$\begin{array}{ccc} i & j & k \\ + & - & + \end{array}$$

$$\begin{vmatrix} 2 & 1 & -3 \\ 1 & 0 & n \\ 4 & 3 & -1 \end{vmatrix} = 0$$

$$+2\begin{bmatrix} 0 & n \\ 3 & -1 \end{bmatrix} - 1\begin{bmatrix} 1 & n \\ 4 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix} =$$

$2(-3n) - 1(-1-4n) - 3(3) = 0$

$-6n + 1 + 4n - 9 = 0$

$1 - 9 = -4n + 6n$

$2n = -8$

$n = -8/2 = -4$

Answer is A

No 9: A unit vector parallel to the resultant of vectors $2i+4j-5k$, $i+2j+3k$ and $-6j+2k$ is A) $-3i$ B) $3i+2j$ C) $3i$ D) $3i-2k$ E) $2i$

SOLUTION

$(2i+4j-5k)+(i+2j+3k)+(-6j+2k)$

$= 3i$

Answer is C

$5-2=c$

BSF9: Evil people Listen to evil ideas and Liars Listen to Lies

VECTOR OR CROSS PRODUCT

The vector product of two non-Collinear Vectors are given as

$$\bar{a} \times \vec{b} = |\bar{a}|\,|\bar{b}| \sin\theta \cdot \hat{\eta}$$

$$\hat{\eta} = \frac{\bar{a} \times \bar{b}}{|\bar{a}|\,|\bar{b}| \sin\theta}$$

$\hat{\eta}$ = unit vector perpendicular to both vector $\bar{a}$ and $\bar{b}$

note:

$\bar{a} \times \bar{b} \neq \bar{b} \times \bar{a}$ i.e Vector product is not comutative but

$$\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$$

If the two vector are parallel $\theta = 0°$

$$\sin 0° = 0$$

$$\bar{a} \times \bar{b} = 0$$

$$i \times i = j \times j = k \times k = 0$$

$$i \times j = -j \times i = k$$

$$j \times k = -k \times i = i$$

$$k \times i = -i \times k = j$$

Vector product can be expressed as the determinant between two vector e.g

$$\bar{a} = a_1 i + a_2 j + a_3 k$$

$$\bar{b} = b_1 i + b_2 j + b_3 k$$

SOLUTION

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

EXAMPLE

solve $\vec{a} \times \bar{b}$ If

$$\bar{a} = i + 2j + 3k$$

$$\bar{b} = 2i + j - k$$

SOLUTION

$$\bar{a} \times \bar{b} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= (-2-3)i - (-1-6)j + (1-4)k$$

$$= -5i + 7j - 3k$$

EXAMPLE

Find a unit vector perpendicular to both the vectors

$$\vec{A} = 2i - 3j + k$$

$$\vec{B} = 7i - 5j + k$$

SOLUTION

$\vec{A} \times \vec{B}$ is required

$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 7 & -5 & 1 \end{vmatrix}$$

$$= (-3+5)i - (2-7)j + (-10+21)k$$

$$= 2i + 5j + 11k$$

Unit vector of $\bar{A} \times \bar{B}$

$$= \frac{2i+5j+11k}{\sqrt{4+25+121}} = \frac{2i+5j+11k}{5\sqrt{6}}$$

COLLINEARITY OF VECTORS

Three vectors are said to be collinear when the vector ~~product of~~ their relative position equal zero i.e vector $\vec{A}, \vec{B}, \vec{C}$ lies in the same straight line when

$$\vec{AB} \times \vec{BC} = 0$$

EXAMPLE

Find the value of λ if the vectors

$\vec{A} = -i + 3j + 2k$

$\vec{B} = -4i + 2j - 2k$ and

$\vec{C} = 5i + \lambda j + 10k$

lies on the same straight line

SOLUTION

note: when vector lies on the same straight line they are said to be coll... When they lies on the same plane such vectors are coplanar.

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= -3i - j - 4k$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= 9i + (\lambda - 2)j + 12k$$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} i & j & k \\ -3 & -1 & -4 \\ 9 & \lambda-2 & 12 \end{vmatrix} =$$

$$(-12 + 4\lambda - 8)i - (-36+36)j + (-3\lambda + 6 + 9)k$$

If $\vec{AB} \times \vec{BC} = 0$

Then

$$-12 + 4\lambda - 8 = 0 \rightarrow (i)$$

OR

$$-3\lambda + 6 + 9 = 0 \rightarrow (ii)$$

Using eq (i) or (ii)

$$4\lambda = 8 + 12$$

$$4\lambda = 20$$

$$\lambda = \frac{20}{4} = 5$$

CHOP → BREAD

If $\bar{a} = 3i - 2j - 2k$ and $\bar{b} = 2i + 3j + k$ Calculate $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$

B.M.F 10: What a joy it is to find just the right word for the right occasion!

CHOP — SALAD

a) What is the unit vector perpendicular to the plane $\bar{a}$ and $\bar{b}$ if $\bar{a} = 4i + 3j + k$ and $\bar{b} = 2i - j + 2k$

HINT: $\bar{a} \times \bar{b}$ - unit vector

b) If $\bar{A} = 2i + 2j - k$, $\bar{B} = 6i - 3j + 2k$. Find the unit vector perpendicular to $\bar{A}$ and $\bar{B}$. Also find the sine of angle between $\bar{A}$ and $\bar{B}$

HINT: $\bar{A} \times \bar{B}$ unit vector

$$\sin\theta = \frac{\bar{A} \times \bar{B}}{|A| \cdot |B|} \text{ times } \frac{1}{\hat{\eta}}$$

$\hat{\eta}$ = unit vector perpendicular to both $\bar{A}$ and $\bar{B}$

$\hat{\eta} = \bar{A} \times \bar{B}$ unit vector

ANSWER

a) $\frac{7i - 6j - 10k}{\sqrt{185}}$

b) (i) $\frac{1}{5\sqrt{17}}(i - 10j - 18k)$

(ii) $\sin\theta = \frac{5\sqrt{17}}{21}$

KEEP MOVING BUT DON'T BE MOVED!!!

APPLICATION OF VECTOR PRODUCTS

A) VECTORIAL AREA OF TRIANGLE

If vector $\vec{A}$, $\vec{B}$ and $\bar{C}$ are the vertices of triangle

$$\text{Area of } \triangle ABC = \frac{1}{2}(\vec{BC} \times \vec{BA})$$

EXAMPLE

Find the area of the triangle formed by the points whose position are $3i + j$, $5i + 2j + k$, $i - 2j + 3k$

SOLUTION

$\vec{A} = 3i + j$ $\quad \vec{B} = 5i + 2j + k$

$\vec{C} = i - 2j + 3k$

$\vec{BC} = \vec{OC} - \vec{OB} = -4i - 4j + 2k$

$\vec{BA} = \vec{OA} - \vec{OB} = -2i - j - k$

$$\vec{BC} \times \vec{BA} = \begin{vmatrix} i & j & k \\ -4 & -4 & 2 \\ -2 & -1 & -1 \end{vmatrix}$$

$= (4+2)i - (4+4)j + (4-8)k$

$= 6i - 8j - 4k$

$\frac{1}{2}(\vec{BC} \times \vec{BA}) = \frac{1}{2}(6i - 8j - 4k)$

$= 3i - 4j - 2k$

magnitude of the area =

$\left|\frac{1}{2}(\vec{BC} \times \vec{BA})\right| = \sqrt{3^2 + (-4)^2 + (-2)^2}$

$= \sqrt{9 + 16 + 4}$

$= \sqrt{29}$ units²

B) VECTORIAL AREA OF PARALLELOGRAM.

The area of parallelogram whose sides is represented by $\vec{A}$ and $\bar{B}$ is given as

Vector area of llgm $= \bar{A} \times \bar{B}$

Example

Find the area of a parallelogram whose adjacent sides are $i - 2j + 3k$ and $2i + j - 4k$

SOLUTION

Vector area of llgm $= \bar{A} \times \bar{B}$

$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{vmatrix}$$

$$= (8-3)i - (-4-6)j + (1+4)k$$

$$= 5i + 10j + 5k$$

Its magnitude $= \sqrt{5^2 + 10^2 + 5^2}$

$$= \sqrt{25 + 100 + 25}$$

$$= \sqrt{150}$$

$$= \sqrt{25 \times 6}$$

$$= 5\sqrt{6} \text{ Unit sq}$$

C) LINEAR VELOCITY

$$\bar{V} = \bar{w} \times \bar{r}$$

EXAMPLE

A rigid body is spinning with an angular velocity of $4 rads^{-1}$ about an axis parallel to $3j + k$ passing through the point $i + 3j - k$. Find the speed of the particles at the point $4i - 2j + k$

SOLUTION

$$\bar{V} = \bar{w} \times \bar{r}$$

$$\vec{A} = 3j + k$$

$$\vec{B} = i + 3j - k$$

$$\vec{P} = 4i - 2j + k$$

$$\vec{AB} = \vec{B} - \vec{A} = i - 2k$$

$\bar{w} = 4 rads^{-1}$ times Unit vector $\vec{AB}$

$$\bar{w} = 4 \frac{(i - 2k)}{\sqrt{1 + (2)^2}} = \frac{4(i - 2k)}{\sqrt{5}}$$

$$\bar{r} = \vec{AP} = \vec{P} - \vec{A}$$

$$\bar{r} = 4i - 5j$$

$$\bar{V} = \bar{w} \times \bar{r}$$

$$\bar{V} = \frac{4}{\sqrt{5}}(i - 2k) \times 4i - 5j$$

$$\bar{V} = \frac{4}{\sqrt{5}} \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 4 & -5 & 0 \end{vmatrix}$$

$$\bar{V} = \frac{4}{\sqrt{5}} \left[(-10)i - (8)j (-5)k \right]$$

$$\bar{V} = \frac{-4}{\sqrt{5}} (10i + 8j + 5k) \text{ Unit}$$

BSF 11: A gentle answer quietens anger, but a harsh one stirs it up.

EXAMPLE

A rigid body is rotating with angular velocity $2 rads^{-1}$ about an axis is $2i-2j+k$ which passes through the origin. Find the velocity of the point $3i+2j-k$ on the body

SOLUTION

$\bar{V} = \bar{w} \times \bar{r}$

$\bar{r} = 3i+2j-k$

$\vec{A} = 2i-2j+k$

$\bar{w} = 2$ times unit vector $\bar{A}$

$$\bar{w} = 2\frac{(2i-2j+k)}{\sqrt{2^2+(-2)^2+1^2}}$$

$$\bar{w} = \frac{2}{3}(2i-2j+k)$$

$\bar{V} = \bar{w} \times r$

$$\bar{V} = \frac{2}{3}(2i-2j+k) \times 3i+2j-k$$

$$\bar{V} = \frac{2}{3}\begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= \frac{2}{3}(5i+10k)$$

Q3

EXAMPLE

A body is rotating with an angular velocity of $2 rads^{-1}$ about the line joining $(1,2,3)$ and $(2,3,5)$. Find the velocity of the particle of the body which is momentarily at $(3,5,6)$

SOLUTION

$\bar{V} = \bar{w} \times \bar{r}$

$A = i+2j+3k$

$B = 2i+3j+5k$

$P = 3i+5j+6k$

$\vec{AB} = \vec{OB} - \vec{OA} = j+2k$

$\vec{w} = 2$ times unit vector $\vec{AB}$

$$\bar{w} = 2\frac{(j+2k)}{\sqrt{1^2+2^2}} = 2\frac{(j+2k)}{\sqrt{5}}$$

$\vec{r} = \vec{P} - \vec{A} = 2i+3j+3k$

$\bar{V} = \bar{w} \times \bar{r}$

$$\bar{V} = 2\frac{(j+2k)}{\sqrt{5}} \times (2i+3j+3k)$$

$$\bar{V} = \frac{2}{\sqrt{5}}\begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2 & 3 & 3 \end{vmatrix}$$

$$= \frac{2}{\sqrt{5}}[(3-6)i - (-4)j - 2k]$$

$$= \frac{2}{\sqrt{5}}(-3i+4j-2k)$$

CHOP → CRAYFISH

9) A rigid body is rotating at the rate $2\frac{1}{2}$ rads^{-1} about an axis of points $(1, -2, 1)$ and $(3, -4, 2)$. Find the velocity of the point $(5, -1, -1)$ on the body

Answer $= \frac{5}{6}(3i + 8j + 10k)$

5) A rigid body is spinning with speed of 45 r.p.m about an axis $(4, -3, 9)$. If the direction cosines of rotation are proportional to 3, −4 and 12. Find the linear velocity of the particle at $(2, -1, 1)$ at the rigid body.

HINT $\vec{AB} = (3i, -4j + 12k)$

$\vec{P} = (2i - j + k)$

$\bar{A} = (4i, -3j + 9k)$

$\bar{r} = \bar{P} - \bar{A}$

$\bar{w}$ = Angular Vel times unit vector $\vec{AB}$

Express angular speed in π

Answer is $\frac{180\pi}{13}(4i - k)$

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II) MOMENT OF A FORCE

Moment $= \bar{r} \times \bar{F}$

note $\bar{r} \times \bar{F} \neq \bar{F} \times \bar{r}$

EXAMPLE

Forces $2i - 5j + 6k$, $-i + 2j - k$ and $2i + 7j$ acts on a points P whose position vector is $4i - 3j - 2k$. Find the vector moment of the resultant of the three forces acting at about point Q $(6, 1, -3)$

SOLUTION

$F = F_1 + F_2 + F_3$

$F = (2i - 5j + 6k) + (-i + 2j - k) + (2i + 7j)$

$F = 3i + 4j + 5k$

Moment $= \bar{r} \times F$

$\bar{r} = (6i + j - 3k) - (4i - 3j - 2k)$

Moment $= (2i + 4j - k) \times (3i + 4j + 5k)$

$$= \begin{vmatrix} i & j & k \\ 2 & 4 & -1 \\ 3 & 4 & 5 \end{vmatrix}$$

~~$= (5+12)i - (30+9)j + (24-2)$~~

~~$= 17i$~~

$= (20+4)i - (13j) + (8-12)k$

$= 24i - 13j - 4k$

Its magnitude

$= \sqrt{24^2 + (-13)^2 + (-4)^2}$

BST 12: people learn from one another just as Iron sharpens Iron.

F) VOLUME OF PARALLELOPIPED

The Volume of parallelopiped is given by the area of a parallelogram times perpendicular distance.

If a parallelopiped has $\bar{a}$, $\bar{b}$, $\bar{c}$ as its Conterminous edges its Volume is obtained by what is called TRIPPLE PRODUCT

$$Vol = \bar{a} \cdot (\bar{b} \times \bar{c})$$

$\bar{b} \times \bar{c}$ = area of parallelogram

NOTE

If $\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$ we can simply says that the Conterminous edges of the parallelopiped are COPLANAR i.e Lies on the same plane

EXAMPLE

Find the Volume of parallelopiped if

$\bar{a} = -3i + 7j + 5K$

$\bar{b} = -3i + 7j - 3K$ 28

$\bar{c} = 7i - 5j - 3K$

SOLUTION

$$Volume = \bar{a} \cdot (\bar{b} \times \bar{c})$$

evaluate $\bar{b} \times \bar{c}$ first

$$\bar{b} \times \bar{c} = \begin{vmatrix} i & j & k \\ -3 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

~~$\bar{b} \times \bar{c} = (-21 + 3)i - (9 + 3)$~~

$\bar{b} \times \bar{c} = (-21 - 15)i - (9 + 21)j + (15 - 49)k$

$\bar{b} \times \bar{c} = -36i - 30j - 34k$

Then

$\bar{a} \cdot (\bar{b} \times \bar{c}) =$

$(-3i + 7j + 5k) \cdot (-36i - 30j - 34k)$

$= 108 - 210 - 170 = -272$

Volume = 272 Unit

ALLITER

determinant method

$$Volume = \begin{vmatrix} -3 & 7 & 5 \\ -3 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$= -3(-21 - 15) - 7(9 + 21) + 5(15 - 49)$

$= 108 - 210 - 170$

$= -272$

$= 272$ Unit

SOLVED PAST QUESTION

2010 No 21: The volume of a parallelepiped whose edges are given by

$a = 2i - 3j + 4k$

$b = i + 2j - k$

$c = 3i - j + 2k$ is

A) 2 B) 21 C) none D) 49 E) 7

SOLUTION

$$\text{Volume} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 2[4-1] + 3[\cancel{2}+3] + 4[-1-6]$$

$$= 2(3) + 3(6) + 4(-7)$$

$$6 + 18 - 28$$

$$24 - 28 = -4$$

none

Answer is C

EQUATION OF A PLANE

To find equation of plane represented by 3 given coordinate (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3) we say

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

2010 No 21: Find the equation of the plane that contains A (4, 1, 2) B (1, 5, 4) and C (-3, 2, (

A) $14x - 2y + 25z = 104$

B) $-14x + 2y - 25z = 10$

C) $-14x + 2y - 25z = 104$

D) $-21x + 7y - 5z = 10$

E) $-14x + 22y - 125z = 104$

SOLUTION

$$\begin{vmatrix} x-4 & y-1 & z-2 \\ -3 & 4 & 2 \\ -7 & 1 & 4 \end{vmatrix} = 0$$

$$(x-4)(16-2) - (y-1)(-12+14) + (z-2)(-3+28) = 0$$

$$14(x-4) - 2(y-1) + 25(z-2) = 0$$

$$14x - 56 - 2y + 2 + 25z - 50 = 0$$

$$14x - 2y + 25z = 50 - 2 + 56$$

$$14x - 2y + 25z = 104$$

Answer is A

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KEEP READING !!!

BSF 13: Good people think before they answer. Evil people have a quick reply but it causes trouble

EXTRA BONUS

Show that the vector

$\bar{a} = 5i + 6j + 7k$

$\bar{b} = 7i - 8j + 9k$

$\bar{c} = 3i + 20j + 5k$ are coplanar

SOLUTION

By triple product

$\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$ for $\bar{a}, \bar{b}, \bar{c}$ to be coplanar

$$\bar{a} \cdot (\bar{b} \times \bar{c}) = \begin{vmatrix} 5 & 6 & 7 \\ 7 & -8 & 9 \\ 3 & 20 & 5 \end{vmatrix}$$

$5(-40 - 180) - 6(35 - 27) + 7(140 + 24) = 0$

$-5 \times 220 - 6 \times 8 + 7 \times 164 =$

$-1100 - 48 + 1148 = 0$

Find the direction cosine joining the line of point $(3, 2, -4)$ and $(1, -1, 2)$

SOLUTION

$\bar{A} = 3i + 2j - 4k$

$\vec{B} = (i - j + 2k)$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= -2i - 3j + 6k$

Unit vector of $\vec{AB}$ =

$$\frac{-2i - 3j + 6k}{\pm\sqrt{(-2)^2 + (-3)^2 + 6^2}} = \frac{-2i - 3j + 6k}{\pm\sqrt{49}}$$

$$= \frac{-2i - 3j + 6k}{\pm 7}$$

direction cosine are given by $2/7, 3/7, \frac{-6}{7}$ or $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$

Find the acute angles which the line joining the points $(1, -3, 2)$ and $(3, -5, 1)$ makes with the coordinates

SOLUTION

$\bar{A} = i - 3j + 2k$

$\bar{B} = 3i - 5j + k$

$\vec{AB} = 2i - 2j - k$

Unit vector $= \frac{2i - 2j + k}{\sqrt{2^2 + (-2)^2 + 1^2}}$

$a)$

Unit vector of $\vec{OB} = \frac{2i-2j+k}{3}$

Acute angle are $\cos^{-1} 2/3$, $\cos^{-1} -2/3$, $\cos^{-1} -1/3$

Two sides of a triangle are formed by the vector $A = 3i+6j-2k$ and $B = 4i-j+3k$. Determine the angle of the triangle

SOLUTION

$\vec{A+B} = (3i+6j-2k)+(4i-j+3k)$
$= 7i+5j+1k$

Unit vector $= \frac{7i+5j+1k}{\sqrt{7^2+5^2+1^2}}$

Unit vector $\frac{7i+5j+1k}{\sqrt{75}}$

$\vec{AB} = i-7j+5k$

Unit vector $= \frac{i-7j+5k}{\sqrt{75}}$

$r = \sqrt{75} \quad r^2 = 75$
$x = 7$

The angles are
$\cos^{-1}\frac{x}{r^2}$, $\cos^{-1}\sqrt{\frac{r^2-x^2}{r^2}}$ $\sin^{-1}\frac{r^2}{r^2}$

Angles are
$\cos^{-1} 7/75$, $\cos^{-1}\frac{\sqrt{26}}{\sqrt{75}}$

SOLVED PAST QUESTIO
No 18 (2010). If $\bar{a} = 4i-j+3k$ and $\bar{b} = -2i+j-k$ then a unit vector perpendicular to both a and b is

A) $\pm\frac{1}{\sqrt{3}}(i+j-k)$
B) $\pm(i-2j-2k)$
C) $\pm3(i-2j-2k)$
E) $\frac{1}{3}(3j-k)$

SOLUTION

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 4 & -1 & 3 \\ -2 & 1 & -1 \end{vmatrix}$$

$= (1-3)i-(-4+6)j+(4-2)k$

$= -2i-2j+2k$

$= 2(-i-j+k)$

Unit Vector

$= \frac{2}{\sqrt{3}}(-i-j+k)$

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No answer.

BSF 14: What you say can preserve life or destroy it. So you must accept the consequence of your words.

EXAMPLE

Find the moment about a line through the origin having the direction of $2i-2j+k$ due to a 30kg force acting at a point $(-4, 2, 5)$ in the direction of $12i-4j-3k$

SOLUTION

$$\text{Force} = 30\text{kg times } \frac{2i-2j+k}{\sqrt{2^2+(-2)^2+1^2}}$$

$$\text{force} = 10(2i-2j+k)$$

$$\bar{r} = -4i+2j+5k$$

$$\text{Moment} = \bar{r} \times \bar{F}$$

$$= (-4i+2j+5k) \times 10(2i-2j+k)$$

$$\text{Moment} = 10\begin{vmatrix} i & j & k \\ -4 & 2 & 5 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= 10[(2+10)i + 14j + 4k]$$

Unit moment in the direction of $12i-4j-3k$

$$= 10(12i+14j+4k) \cdot \frac{(12i-4j-3k)}{\sqrt{12^2+(-4)^2+(-3)^2}}$$

$$\text{Unit moment} = \frac{10}{13}(144-56-12)$$

$$= \frac{760}{13} \text{ Unit}$$

EXAMPLE

A force is represented in magnitude and direction by the line joining the point $A(1,-2,4)$ to the point $B(5,2,3)$. Find its moment about the point $(-2,3,5)$.

SOLUTION

$$\text{Force} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= (5i+2j+3k) - (i-2j+4k)$$

$$\vec{F} = 4i+4j-k$$

$$\bar{r} = \overset{\overrightarrow{OP}}{(-2i+3j+5k)} - \overset{\overrightarrow{OA}}{(i-2j+4k)}$$

$$= -3i+5j+1k$$

$$\text{Moment} = \bar{r} \times \bar{F}$$

$$= \begin{vmatrix} i & j & k \\ -3 & 5 & 1 \\ 4 & 4 & -1 \end{vmatrix}$$

$$(-5-4)i - (3-4)j + (-12-20)$$

$$= -9i + j - 32k$$

Magnitude

$$= \sqrt{(-9)^2+1^2+(-32)^2}$$

$\Rightarrow$? ?

CHOP - CHIN CHIN

Caculate the moment about the point of a force (~~#~~, 1, 1) of 5Kg acting along the Line $\vec{AB}$ where A and B are points (2,3,4) and (3, 5, 6)

~~[illegible]~~

A Force $F = 3i + 2j - 4K$ is applied at the point (1, -1, 2). Find the moment about (2, -1, 3)

HINT: (a) F = 30 times $\frac{i + j + k}{\sqrt{3}}$

$r = (3i + 5j + 6k) - (2i + 3j + 4k)$

b) $r = (2i - j + 3j) - (i - j + 2k)$

SOLVED PAST QUESTION

2008/2009: The area of the triangle having the vertices of (1, 3, 2), (2, -1, 1) and (-1, 2, 3) is A) $\sqrt{107}$ B) $2\sqrt{107}$ C) 11 D) $\frac{\sqrt{107}}{2}$ E) $\frac{2\sqrt{107}}{3}$

SOLUTION

$\bar{A} = i + 3j + 2K$

$\bar{B} = 2i - j + K$

$\bar{C} = -i + 2j + 3K$

$\vec{BC} = (-i + 2j + 3K) - (2i - j + k)$

$= -3i + 3j + 2K$

$\vec{BA} = (i + 3j + 2k) - (2i - j + k)$

$\vec{BA} = -i + 4j + k$

Area of $\Delta ABC = \frac{1}{2}|\vec{BC} \times \vec{BA}|$

$$\vec{BC} \times \vec{BA} = \begin{vmatrix} i & j & K \\ -3 & 3 & 2 \\ -1 & 4 & 1 \end{vmatrix}$$

$\vec{BC} \times \vec{BA} = (3-8)i - (-3+2)j + (-12+3)k$

$= -5i + j - 9K$

$\frac{1}{2}(\vec{BC} \times \vec{BA}) = \frac{1}{2}(-5i + j - 9k)$

Its magnitudes

$= \frac{1}{2}\sqrt{(-5)^2 + 1^2 + (-9)^2}$

$= \frac{1}{2}\sqrt{25 + 1 + 81} = \frac{\sqrt{107}}{2} = 5.2$

Answer is D

ALLITER COMPARISON

Area = half of determinant of the coordinate of the vertices

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ -1 & 2 & 3 \end{vmatrix}$$

$= \frac{1}{2}[1(-3-2) - 3(6+1) + 2(4-1)]$

$= \frac{1}{2}[-6 - 3(7) + 2(3)]$

$= \frac{1}{2}[-6 - 21 + 6] = \underline{10.5}$

Note: Vectorial area $\neq$ 3D Coordinate area

BSF 14: A hard-working farmer has plenty to eat. people who waste time will always be poor.

2007/2008 No 22: A unit vector perpendicular to both $3i + j - 2k$ and $2i - 3j - k$ is A) $\frac{7i - j + 11k}{\sqrt{171}}$

B) $\frac{7i + j - 11k}{\sqrt{171}}$ C) $\frac{7i + j + 11k}{\sqrt{177}}$

D) $\frac{17i + j + 11k}{\sqrt{171}}$ E) $\frac{7i + j + 11k}{\sqrt{171}}$

SOLUTION

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 2 & -3 & -1 \end{vmatrix}$$

$= (-1 - 6)i - (-3 + 4)j + (-9 - 2)k$

$= -7i - j - 11k$

$$\text{unit vector} = \frac{-7i - j - 11k}{\pm\sqrt{(-7)^2 + (-1)^2 + (-11)^2}}$$

$$\text{unit vector} = -\frac{(7i + j + 11k)}{\sqrt{171}}$$

or

$\frac{7i + j + 11k}{\sqrt{171}}$ Answer is E

2007/2008 No 19: The force given by $2i - j + k$ acted through the point $i + j + k$. Find the magnitude of the moment of the force about the point $i - j + k$ A) $\sqrt{5}$ B) $-2\sqrt{5}$ C) $3\sqrt{5}$ D) $-\sqrt{5}$ E) $2\sqrt{5}$

SOLUTION

$\bar{F} = 2i - j + k$

$\bar{B} = i + j + k$

$\bar{A} = i - j + k$

$\bar{r} = \bar{OB} - \bar{OA} = +2j$

Moment $= \bar{r} \times \bar{F}$

$$= \begin{vmatrix} i & j & k \\ 0 & +2 & 0 \\ 2 & -1 & 1 \end{vmatrix}$$

$= 0$

No answer

2009: No 35: A force $F = 3i + 2j - 4k$ is applied at the point $(1, -1, 2)$. find the moment of the force about the point $(2, -1, 3)$

A) $-2i + 7j + 2k$ B) $\frac{5}{6}(3i + 7j + \ldots$

C) $2i - 3j + 6k$ D) $3i + 2j - 8k$

E) $2i - 7j - 3k$

SOLUTION

$\bar{F} = 3i + 2j - 4k$

$\bar{B} = \hat{i} - j + 2k$

$\bar{A} = 2i - j + 3k$

$\bar{r} = \bar{B} - \bar{A} = -(i + k)$

Moment $= \bar{r} \times \bar{F}$

$$\begin{vmatrix} i & j & k \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$= (0+2)i - (4+3)j + (-2-0)k$

$= 2i - 7j - 2k$

In anticlockwise direction

moment $= -(2i - 7j - 2k)$

$= -2i + 7j + 2k$

Answer is A

2010 : No 16: A rigid body is spinning with angular velocity of $4 rads^{-1}$ about an axis parallel to $3i - k$ passing through the point $i + 3j - k$. The velocity of the particle at the point $4i + 2j + k$ is given by

A) $(i - 3j - 9k)$ B) $4(i - 3j + 9k)$

C) $(i + j + k)$ D) $\frac{4}{\sqrt{10}}(i - 3j - 9k)$

E) $\frac{4}{\sqrt{10}}(3i - k)$

SOLUTION

$\bar{w} = 4$ times $\frac{3i - k}{\sqrt{3^2 + (-1)^2}}$

$\bar{w} = \frac{4}{\sqrt{10}}(3i - k)$

$\bar{V} = \bar{w} \times \bar{r}$

$\bar{r} = (4i + 2j + k) - (i + 3j - k)$

$\bar{r} = 3i - j + 2k$

$\bar{V} = \frac{4}{\sqrt{10}}(3i - k) \times (3i - j + 2k)$

$$\bar{V} = \frac{4}{\sqrt{10}} \begin{vmatrix} i & j & k \\ 3 & 0 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= \frac{4}{\sqrt{10}}(i - 9j - 3k)$

No answer

No 34: A plane is perpendicular to the vector $2i + 3j + 6k$ passes through the terminal points of the vector $i + 5j + 3k$. Find the distance of the plane to the origin A) 35 B) 5 C) 7 D) 33 E) -5

SOLUTION

$(2i + 3j + 6k) \cdot (i + 5j + 3k)$

$2 + 15 + 18 = \underline{35}$

Answer is A

$$\frac{dr}{dt} = 0 - j + 2k = -j + 2k$$

$$\frac{d^2r}{dt^2} = 2i - 0 + 0$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2} = 2$$

EXAMPLE

$\bar{a} = t^3\bar{i} + t^2\bar{j} + t\bar{k}$

$\bar{b} = (t+1)i + (t+2)j - 3t\bar{k}$

$$\bar{a}\cdot\bar{b} = t^3(t+1) + t^2(t+2) - t(3t)$$

$$= t^4 + t^3 + t^3 + 2t^2 - 3t^2$$

$$= t^4 + 2t^3 - t^2$$

$$\frac{d}{dt}(\bar{a}\cdot\bar{b}) = 4t^3 + 6t^2 - 2t$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ t^3 & t^2 & t \\ (t+1) & (t+2) & -3t \end{vmatrix}$$

$$\bar{a}\times\bar{b} = [-3t^3 - t(t+2)]i - [t^3(-3t) - t(t+1)]j + [t^3(t+2) - t^2(t+1)]j$$

$$\bar{a}\times\bar{b} = (-3t^3 - t^2 - 2t)i - (-3t^4 - t^2 - t)j + (t^4 + 2t^3 - t^3 - 2t^2)j$$

$$\frac{d}{dt}(\bar{a}\times\bar{b}) = (-9t^2 - 2t - 2)i - (-12t^3 - 2t - 1)j + (4t^3 + 6t^2 - 3t^2 - 4t)k$$

$$\frac{d}{dt}(\bar{a}\times\bar{b}) = -(9t^2 + 2t + 2)i + (12t^3 + 2t + 1)j + (4t^3 + 3t^2 - 4t)k$$

TANGENT AND NORMAL VECTOR

$x = 3\cos t,\ y = 3\sin t,\ z = 4t$

find (a) Tangent vector
(b) Unit tangent vector
(c) Normal vector
(d) Unit Normal vector

SOLUTION

$\bar{r} = xi + yj + zk$

$\bar{r} = 3\cos t\, i + 3\sin t\, j + 4t\, k$

$$\frac{dr}{dt} = -3\sin t\, i + 3\cos t\, j + 4k$$

(a)

tangent vector $= -3\sin t\, i + 3\cos t\, j + 4k$

(b)

Unit tangent vector $=$

$$\frac{-3\sin t\, i + 3\cos t\, j + 4k}{\sqrt{(-3\sin t)^2 + (3\cos t)^2 + 4^2}} = \frac{-3\sin t\, i + 3\cos t\, j + 4k}{\sqrt{9\sin^2 t + 9\cos^2 t + 4^2}} = \frac{-3\sin t\,\hat{i} + 3\cos t\, j + 4k}{\sqrt{9(\sin^2 t + \cos^2 t) + 16}} = \frac{-3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 4k}{\sqrt{9 + 16}} =$$

BSF15: Anyone who loves knowledge wants to be told when he is wrong. It is stupid to hate being corrected.

DIFFRENTATION OF VECTOR

1) $\frac{d}{dt}(\bar{a} \pm \bar{b}) = \frac{d\bar{a}}{dt} \pm \frac{d\bar{b}}{dt}$

2) $\frac{d}{dt}(\bar{a} \cdot \bar{b}) = \bar{a} \cdot \frac{d\bar{b}}{dt} + \frac{d\bar{a}}{dt} \cdot \bar{b}$

③ $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{a} \times \frac{d\bar{b}}{dt} + \frac{d\bar{a}}{dt} \times \bar{b}$

④ $\frac{d}{dt}(\phi \bar{a}) = \phi \frac{d\bar{a}}{dt} + \frac{d\phi}{dt}\bar{a}$

ϕ is a scalar

EXAMPLE

If vector $\bar{r} = \bar{a}\cos wt + \bar{b}\sin wt$

prove that ① $\bar{r} \times \frac{dr}{dt} = w(\bar{a} \times \bar{b})$

⑪ $\frac{d^2\vec{r}}{dt^2} = -wr$ where $\bar{w}$

$\bar{a}, \bar{b}$ are constant

SOLUTION

$\bar{r} = \bar{a}\cos wt + \bar{b}\sin wt$

$\frac{d\bar{r}}{dt} = -w\bar{a}\sin wt + w\bar{b}\cos wt$

$\bar{r} \times \frac{dr}{dt} = (\bar{a}\cos wt + \bar{b}\sin wt) \times (-w\bar{a}\sin wt + w\bar{b}\cos wt)$

$=$ 34

$\cancel{\bar{a}\times\bar{a}}\, w\sin wt\cos wt + \bar{a}\times\bar{b}\cos^2 w$

$\cancel{-\bar{b}\times\bar{a}}\, w\sin^2 wt + w\bar{b}\times\bar{b}$

$\sin wt \cos wt$

note $-\bar{b}\times\bar{a} = \bar{a}\times\bar{b}$

$\bar{b}\times\bar{b} = 0$

$\bar{a}\times\bar{a} = 0$

$\therefore = \bar{a}\times\bar{b}w\cos^2 wt + \bar{a}\times\bar{b}w\sin^2 wt$

$= (\bar{a}\times\bar{b})w(\cos^2 wt + \sin^2 wt)$

$= w(\bar{a}\times\bar{b})(1)$

$= w(\bar{a}\times\bar{b})$

⑪

$\bar{r} = \bar{a}\cos wt + \bar{b}\sin wt$

$\frac{d\vec{r}}{dt} = -w\bar{a}\sin wt + w\bar{b}\cos wt$

$\frac{d\bar{r}^2}{dt^2} = -w^2 a\cos wt - w^2\bar{b}\sin wt$

$\frac{d\bar{r}^2}{dt^2} = -w^2(\bar{a}\cos wt + \bar{b}\sin wt)$

where $r = a\cos wt + b\sin wt$

$\frac{d\bar{r}^2}{dt^2} = -w^2 r$

EXAMPLE

If $\bar{r} = t^2\bar{i} - t\bar{j} + (2t+1)k$

find at $t=0$ the values of $\frac{dr}{dt}$ and $\left|\frac{d^2r}{dt^2}\right|$

SOLUTION

$r = t^2 i - tj + (2t+1)k$

$\frac{dr}{dt} = 2t - j + 2k$

at $t=2$

16: Sensible people will see trouble coming and avoid it, but an unthinking person will walk right into it and regret it later.

$= \frac{1}{5}(-3\sin t + 3\cos t + 4K)$

(c)

Normal Vector =

$$\frac{\frac{d^2r}{dt^2}}{\left|\frac{dr}{dt}\right|} = \frac{\frac{1}{5}(-3\cos t - 3\sin t)}{\sqrt{(-3\sin t)^2 + (3\cos t)^2 + 4^2}}$$

Normal Vector =

$$\frac{\frac{1}{5}(-3\cos t\,i - 3\sin t\,j)}{5} =$$

$$\frac{1}{25}(-3\cos t\,i - 3\sin t\,j)$$

(d)

Unit Normal Vector =

$$\frac{\frac{1}{25}(-3\cos t\,i - 3\sin t\,j)}{\frac{1}{25}\sqrt{(-3\cos t)^2 + (3\sin t)^2}}$$

$$= \frac{1/25(-3\cos t\,i - 3\sin t\,j)}{\frac{1}{25}\sqrt{9(\cos^2 t + \sin^2 t)}}$$

$$= \frac{1/25(-3\cos t\,i - 3\sin t\,j)}{3/25}$$

$$= \frac{\frac{3}{25}(-\cos t\,i - \sin t\,j)}{3/25}$$

$= -\cos t\,i - \sin t\,j$

SOLVED PAST QUESTION

2010 No 9: Suppose a and b are vector function of scalar t, then which of the following is NOT TRUE

A) $\frac{d(a+b)}{dt} = \frac{da}{dt} + \frac{db}{dt}$

B) $\frac{d(a \cdot b)}{dt} = a \cdot \frac{db}{dt} + b \cdot \frac{da}{dt}$

C) $\frac{d(a \times b)}{dt} = a \times \frac{db}{dt} + b \times \frac{da}{dt}$

D) $\frac{d(a+b)}{dt} = \frac{da}{dt} \times \frac{db}{dt}$

E) $\frac{d(a-b)}{dt} = \frac{da}{dt} - \frac{db}{dt}$

SOLUTION

Both C AND D are wrong but preferably we choose D

Check Diffrentiation of Vector

No 10: Let a be vector function of scalar t, the necessary and sufficient condition for a to be constant is A) $\frac{da}{dt} = 0$ (null vector) B) $a \cdot \frac{da}{dt} = 0$

C) $a \times \frac{da}{dt} = 0$

D) $\frac{d(a+b)}{dt} = 0$ E) $\frac{d(a-b)}{dt} = 0$

SOLUTION

No 11: If $a = 5t^2 i - tj - t^3 k$ and $b = \sin t\, i - \cos t\, j$. Then

$a \cdot \frac{db}{dt} =$

A) $(5t-1)\cos t$ B) $(5t^2-1)\sin t$
C) $5t^2\cos t + t\sin t$ D) $(5t-1)\sin t$
E) $5t^2 \sin t$

SOLUTION

$b = \sin t\, i - \cos t\, j$

$\frac{db}{dt} = \cos t\, i + \sin t\, j + 0k$

$a = 5t^2 i - tj - t^3 k$

$a \cdot \frac{db}{dt} = 5t^2 \cos t - t\sin t$

No Answer But choose C

2007/2008 NO 22: If
$\bar{r} = t^2 i - tj + (2t+1)\bar{k}$.
What is the value of
$\left|\frac{d^2r}{dt^2}\right|$? A) 1 B) 2 C) 3 D) -2

SOLUTION

Answer is B: check example

No 23: A particle moves so that its position vector is given by $\bar{r} = \cos wt\, i + \sin wt\, j$ where w is a constant. Find the acceleration of the particle. A) w^2 B) $2w^2$ C) $-w^2 r$ C) $w^2 r$ E) w

SOLUTION

$accel = \frac{d^2\bar{r}}{dt} = -w^2 r$

Check Example

Answer is C

BSF BRAINSTORM QUESTION

The tangent vector to the curve whose parametric equation is $x = t^3$, $y = \frac{t+1}{t}$, $z = t^2+1$ at $t = 2$ is

A) $12i + \frac{1}{4}j - 4k$ B) $12i - \frac{1}{4}j + 4k$

C) $12i - \frac{1}{4}j - 4k$ D) $12i + \frac{1}{4}j + \frac{1}{4}k$

SOLUTION

$\bar{r} = xi + yj + zk$

$\bar{r} = t^3 i + \frac{t+1}{t} j + (t^2+1)k$

$\frac{d\bar{r}}{dt} = 3t^2 i + \left(0 - \frac{1}{t^2}\right)j + 2t\, k$

at $t = 2$

$\frac{d\bar{r}}{dt} = 12i - \frac{1}{4}j + 4k \rightarrow B$

CHOP → INDOMIE

Find the unit tangent and unit normal vector at $t = 2$ on the curve $x = t^2 - 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ where t is any variable

PSF 17: Never boast about tommorow! You don't know what will happen between now and then.

ACCELERATION & VELOCITY

If a particle moves along a curve $x = a\cos t$ $y = a\sin t$ $z = at\tan\alpha$ determine the acceleration and velocity of the particle

SOLUTION

$$\bar{r} = xi + yj + zk$$

$$\bar{r} = a\cos t\, i + a\sin t\, j + at\tan\alpha k$$

$$Vel = \frac{dr}{dt} = -a\sin t\, i + a\cos t\, j + a\tan\alpha\, k$$

$$accel = \frac{d^2r}{dt} = -a\cos t\, i - a\sin t\, j + 0$$

$$|\bar{V}| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + (a\tan\alpha)^2}$$

$$|\bar{V}| = \sqrt{a^2[(\sin^2 t + \cos^2 t) + \tan^2\alpha]}$$

$$|\bar{V}| = \sqrt{a^2(1 + \tan^2\alpha)}$$

$$= \sqrt{a^2 \sec^2\alpha} = a\sec\alpha$$

$$|\bar{a}| = \sqrt{(-a\cos t)^2 + (-a\sin t)^2}$$

$$|\bar{a}| = \sqrt{a^2(\cos^2 t + \sin^2 t)} = a$$

EXAMPLE

A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$. where t is the time. Find the component of its velocity and acceleration at time $t = 1$ in the direction $2i + 3j + 6k$

SOLUTION

$$\bar{r} = xi + yj + zk$$

$$\bar{r} = (t^3+1)i + t^2 j + (2t+5)k$$

$$\frac{d\bar{r}}{dt} = 3t^2 i + 2tj + 2k$$

$$Vel = \frac{d\bar{r}}{dt} \text{ at } t=1 = 3i + 2t + 2k$$

Vel in the direction $2i + 3j + 6k$

$$= (3i + 2t + 2k) \cdot \frac{(2i + 3j + 6k)}{\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{6 + 6 + 12}{7} = \frac{24}{7}$$

$$accel = \frac{d^2\bar{r}}{dt} = 6t\, i + 2j$$

$$accel = \frac{d^2r}{dt} \text{ at } t=1 = 6i + 2j$$

accel in the direction $2i + 3j + 6k$ =

$$(6i + 2j) \cdot \frac{(2i + 3j + 6k)}{\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{12 + 6}{7} = 18/7$$

DRINK HOLLANDIA

1) A particle moves along the curve $x = 2t^2$ $y = t^2 - 4t$ and $z = 3t - 5$ where t is the time. Find the components of its Velocity and acceleration at time $t = 1$ in the direction $i - 3j + 2k$

Answer → $\frac{8\sqrt{14}}{7}$, $-\frac{\sqrt{14}}{7}$

b) The coordinates of a moving particle are given by $x = 4t - \frac{t^2}{2}$ and $y = 3 + 6t - \frac{t^3}{6}$. Find the Velocity and acceleration of the particles at $t = 2$ sec

c) check 2010 NO 12 and 13

INTEGRATION OF VECTORS

(A) WORKDONE

If $\bar{F}$ represents the Variable force acting on a particle along arc AB then the total workdone

$$W = \int_A^B \bar{F} \cdot d\bar{r}$$

EXAMPLE

If a Force $\bar{F} = 2x^2yi + 3xyj$ displaces a particle in the xy-plane from (0,0) to (1,4) along a curve $y = 4x^2$. Find the workdone

SOLUTION

$\bar{r} = xi + yj$ [xy-plane]

$y = 4x^2$

$\bar{r} = xi + 4x^2j$

$d\bar{r} = i + 8xj$

$\vec{F} = 2x^2yi + 3xyj$

$\bar{F} = 2x^2(4x^2)i + 3x(4x^2)j$

$\bar{F} = 8x^4i + 12x^3j$

$W = \int_0^1 \bar{F} \cdot d\bar{r} = \int_0^1 (8x^4i + 12x^3j) \cdot (i + 8xj)$

$W = \int_0^1 8x^4 + 96x^4$

$= 104 \int_0^1 x^4$

$= 104 \left[\frac{x^5}{5}\right]_0^1$

$\frac{104}{5}$

EXAMPLE

Find the total workdone in moving a particle in a Force field given by $\bar{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1$ $y = 2t^2$ $z = t^3$ from $t = 1$ to $t = 2$.

BSF 18: It is better to be patient than powerful. It is better to win control over yourself than over whole cities

SOLUTION

Workdone $= \int_1^2 \bar{F} \cdot d\bar{r}$

If $\bar{r} = xi + yj + zk$

$dr = dx\,i + dy\,j + dz\,k$

Workdone $= \int_1^2 ((3xy)i - 5zj + 10xk) \cdot (dx\,i + dy\,j + dz\,k)$

$x = t^2+1 \quad dx = 2t\,dt$

$y = 2t^2 \quad dy = 4t\,dt$

$z = t^3 \quad dz = 3t^2\,dt$

$W = \int_1^2 3xy\,dx - 5z\,dy + 10x\,dz$

$W = \int_1^2 3(t^2+1)(2t^2)2t - 5t^3(4t) + 10(t^2+1)3t^2$

$W = \int_1^2 12t^5 + 12t^3 - 20t^4 + 30t^4 + 30t^2$

$W = 2t^6 + 3t^4 + 2t^5 + 10t^3 \Big|_1^2$

$W = 2[2^6 - 1^6] + 3[2^4 - 1^4] + 2[2^5 - 1^5] + 10[2^3 - 1^3]$

$W = 2(63) + 3(15) + 2(31) + 10(7)$

$W = 126 + 45 + 62 + 70$

$W = 303J$

SOLVED PAST QUESTION,

2009: NO 38: Find the total workdone in moving a particle in a force field given by $\vec{F} = 3yi - 5zj + 10xk$ along the curve $x = t$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$

A) 107 units B) 10.7 units C) 52.5 units D) 100 units E) -107 units

SOLUTION

Workdone $= \int_1^2 \bar{F} \cdot d\bar{r}$

If $\bar{r} = xi + yj + zk$

$d\bar{r} = dx\,i + dy\,j + dz\,k$

Workdone $= \int_1^2 (3yi - 5zj + 10xk) \cdot (dx\,i + dy\,j + dz\,k)$

$x = t \quad dx = 1\,dt$

$y = 2t^2 \quad dy = 4t\,dt$

$z = t^3 \quad dz = 3t^2\,dt$

$W = \int_1^2 3y\,dx - 5z\,dy + 10x\,dz$

$W = \int_1^2 3(2t^2) - 5t^3(4t) + 10t(3t^2)$

$W = \int_1^2 6t^2 - 20t^4 + 30t^3$

$W = \frac{6}{2}t^3 - 4t^5 + \frac{15}{2}t^4 \Big|_1^2$

$W = \frac{6}{2}[2^3 - 1^3] + \frac{15}{2}[2^4 - 1^4] - 4[2^5 - 1^5]$

$W = 6([illegible]) - 4(31) + \frac{15}{2}(15)$

$W = 3 \times 7 - 124$

BSF BRAINSTORM QUESTION

If a force of magnitude 10 units acts in the direction of $2i+j+k$ displaced a particle from the point (1,2,3) to (4,3,6). Find the workdone on the particle

SOLUTION

$$F = 10 \text{ times } \frac{2i+j+k}{\sqrt{2^2+1^2+1^2}}$$

$$F = \frac{10(2i+j+k)}{\sqrt{6}}$$

$$\bar{r} = (4i+3j+6k)-(i+2j+3k)$$

$$\bar{r} = 3i+j+3k$$

$$\text{Workdone} = \vec{F}\cdot d\vec{r}$$

$$= 10\frac{(2i+j+k)}{\sqrt{6}} \cdot 3i+j+3k$$

$$= \frac{10}{\sqrt{6}}(6+1+3) = \frac{100}{\sqrt{6}} \text{ unit}$$

B) VELOCITY AND DISPLACEMENT

The acceleration of a particle is given as

$$\frac{d^2r}{dt^2} = 12\cos t\, i - 8\sin t\, j + 16tk$$

If the velocity $\bar{v}$ and displacement $\bar{r}$ are zero at $t=0$ find $\bar{v}$ and $\bar{r}$ at any time t

SOLUTION

$$accel = \frac{d^2r}{dt^2} = 12\cos 2t\, i - 8\sin 2t\, j + 16tk$$

$$\bar{V} = \int \bar{a}\,dt = \int \frac{d^2r}{dt^2} = \int 12\cos 2t\, i - 8\sin 2t\, j + 16tk$$

$$\bar{V} = 6\sin 2t\, i + 4\cos 2t\, j + 8t^2k + c$$

at $\bar{V}=0$ when $t=0$

$$0 = 6\sin 0\, i + 4\times\cos 0\, j + 0k + c$$

$$0 = 0i + 4j + 0k + c$$

$$c = -4j$$

$$\bar{V} = 6\sin 2t\, i + (4\cos 2t - 4)j + 8t^2k$$

$$\text{displacement} = \bar{r} = \int \bar{V}\,dt$$

$$\bar{r} = \int 6\sin 2t\, i + (4\cos 2t - 4)j + 8t^2k$$

$$\bar{r} = -3\cos 2t\, i + (2\sin 2t - 4t)j + \frac{8}{3}t^3k$$

at $\bar{r}=0$ and $t=0$

$$0 = -3\cos 0\, i + (2\sin 0 - 0)j + 0k + c$$

$$0 = -3i + 0j + 0k + c$$

$$c = 3i$$

$$\bar{r} = (3\cos 2t + 3)i + (2\sin 2t - 4t)j + \frac{8}{3}t^3k$$

C) DEFINITE INTEGRAL

2020 No 14: If $r(t) = 2i - j + 3k$ when $t=2$ and $\frac{dr}{dt} = 4i - 2j$

MSF19: Never tell your neigbour to wait until tomorrow If you can help him now

No 19: The Value of $\int_0^{\pi/2}(3\sin\theta i + 2\cos\theta j)$

= A) $i+j+k$ B) $3i+2j$ C) $3i+2j+k$

D) $\pm(3i+2j+k$ E) $3i-2j$

SOLUTION

$$\int_0^{\pi/2} 3\sin\theta\, i + 2\cos\theta\, j = -3\cos\theta i + 2\sin\theta j \Big|_{0^\circ}^{90^\circ}$$

$$= -3(\cos 90 - \cos 0)i + 2(\sin 90 - \sin 0)j$$

$$= -3(-1)i + 2(1)j$$

$= 3i + 2j$ Answer is B

No 24: If $r = 5t^2 i + tj - t^3 k$

then $\int_0^2 \left(r \times \frac{d^2r}{dt^2}\right)dt =$

A) $\frac{1}{4}(-4i+20j-8k)$ B) $4(-4i+20j-5k)$

C) $i+j+k$ D) $-14i+75j-15k$ E) none

SOLUTION

$$r = 5t^2 i + tj - t^3 k$$

$$\frac{dr}{dt} = 10ti + j - 3t^2 k$$

$$\frac{d^2r}{dt^2} = 10i - 6tk$$

H1

$$r \times \frac{d^2r}{dt^2} = \begin{vmatrix} i & j & k \\ 5t^2 & t & -t^3 \\ 10 & 0 & -6t \end{vmatrix}$$

$$= -6t^2 i - (-30t^3 + 10t^3)j + (-10t)k$$

$$\int_0^2 \left(r \times \frac{d^2r}{dt^2}\right) = \int_0^2 -6t^2 i + 20t^3 j - 10tk$$

$$= -2t^3 i + 5t^4 j - 5t^2 k \Big|_0^2$$

$$= -2(2^3)i + 5(2^4)j - 5(2)^2 k$$

$$= -2(8)i + 5(16)j - 5(4)k$$

$$= -16i + 80 - 20k$$

$$= 4(-4i + 20 - 5k)$$

Answer is B

2007/2008 No 24: If

$\bar{r} = (t - t^2)i + 2t^3 j - 3k$

find $\int_1^2 \bar{r}(t)\,dt$

A) $\frac{5}{6}i + \frac{15}{2}i - 3\bar{k}$ B) $\frac{-5}{6}i + \frac{15}{2}j - 3\bar{k}$

C) $\frac{-5}{6}i - \frac{15}{2}j + 3\bar{k}$ D) $\frac{-5}{6}i + \frac{15}{2}j + 3k$

E) $\frac{-5}{6}i - \frac{15}{2}j + 3\bar{k}$

SOLUTION

$$\int_1^2 \bar{r}(t)\,dt = \int_1^2 (t - t^2)i + 2t^3 j - 3k$$

$$= \left(\frac{t^2}{2} - \frac{t^3}{3}\right)i + \frac{1}{2}t^4 j - 3t \Big|_1^2$$

$$\left[\frac{1}{2}(2^2 - 1^2) - \frac{1}{3}(2^3 - 1^3)\right]i + \frac{1}{2}(2^4 - 1^4)j$$

$$3(2 - 1)$$

$$(3/2 - 7/3)i + 15/2\, j - 3k$$

$= \frac{9-14}{6}i + \frac{15}{2}j - 3k$

$= \frac{-5i}{6} + \frac{15}{2}j - 3k$

Answer is B

D) INDEFINITE INTEGRATION

2007/2008 No 28 : 1F

$\bar{a} = 5t^2 i - 2tj + k$, $\bar{b} = -2ti + 2tj + tk$

find $\int (a \cdot b) dt$ A) $-10t^4 - 4t^3 + t^2$ B) $\frac{5}{2}t^4 - \frac{4}{3}t^3 + \frac{1}{2}t^2$ C) $-t^3 - 4t^2 + t$ D) $\frac{-5}{2}t^4 + \frac{4}{3}t^3 + \frac{1}{2}t^2$ E) $\frac{-5t^4}{2} - \frac{4t^3}{3} + \frac{1}{2}t^2$

SOLUTION

$\bar{a} \cdot \bar{b} = (5t^2 i - 2tj + k) \cdot (-2ti + 2tj + tk)$

$\bar{a} \cdot \bar{b} = -10t^3 - 4t^2 + t$

$\int (\bar{a} \cdot \bar{b}) dt = \int -10t^3 - 4t^2 + t$

$= \frac{-10t^4}{4} - \frac{4t^3}{3} + \frac{t^2}{2}$

$= -\frac{5}{2}t^4 - \frac{4}{3}t^3 + \frac{1}{2}t^2$

Answer is E

No 25. Given that $F = zi - xj + yk$ and $F = \cos t\, i + \sin t\, j + zk$ evaluate $\int F \cdot \vec{dr}$

A) $\int (t \sin t + \cos^2 t + \sin t)\, dt$

B) $\int (-t \sin^2 t - \cos^2 t + \sin t)\, dt$

C) $\int (-t \sin t + \cos^2 t + \sin t)\, dt$

D) $\int (-t \sin t - \cos^2 t + \sin t)\, dt$

E) $\int (t \sin t - \cos^2 t + \sin t)\, dt$

SOLUTION

$\bar{F} = zi - xj + yk$

$F = xi + yj + zk$

$F = \cos t\, i + \sin t\, j + zk$

by comparison

$x = \cos t$ $y = \sin t$, $z = z(t)$

$d\bar{r} = -\sin t\, i + \cos t\, j + 1k$

$\bar{F} = ti - \cos t\, j + \sin t\, k$

$\bar{F} \cdot \vec{dr} = -t \sin t - \cos^2 t + \sin t$

$\int \bar{F} \cdot d\bar{r} = \int (-t \sin t - \cos^2 t + \sin t)\, dt$

Answer is D

CONCENTRATED HYPER

2009 No 16 & 17. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is time

16). The component of the Velocity at $t = 1$ in the direction $i + j + 3k$ is A) 11 B) $\sqrt{11}$ C) 10 D) 9 E) $\frac{8\sqrt{11}}{11}$

17) The component of the acceleration at time $t = 1$ in the direction $i + j + 3k$ is A) 11 B) $\sqrt{11}$ C) 10 D) 9 E) $8\sqrt{11}/11$

BSF 20: Wise men will gain an honourable reputation but stupid men will only add to their own share.

SOLUTION

16) $r = xi + yj + zk$

$r = (t^3+1)i + t^2 j + (2t+5)k$

$\frac{dr}{dt} = 3t^2 i + 2tj + 2k$

$\left.\frac{dr}{dt}\right|_{t=1} = 3i + 2j + 2k$

Vel in the direction of $i + j + 3k$ =

$(3i+2j+2k) \cdot \frac{i+j+3k}{\sqrt{1^2+1^2+3^2}} = \frac{3+2+6}{\sqrt{11}}$

$= \frac{11}{\sqrt{11}} = \sqrt{11}$

answer is B

17) accel $= \frac{d^2r}{dt^2} = 6ti + 2j + 0k$

$\left.\frac{d^2r}{dt^2}\right|_{at\ t=1} = 6i + 2j + 0k$

accel in the direction of $i+j+3k$ =

$(6i+2j+0k) \cdot \frac{i+j+3k}{\sqrt{1^2+1^2+3^2}} = \frac{6+2}{\sqrt{11}}$

$= \frac{8}{\sqrt{11}} = \frac{8\sqrt{11}}{11}$ Answer is E

No 19: The magnitude of the acceleration of a particle which moves along the curve $x = t^3+1$, $y = t^2$, $z = 2t+5$ at time $t = 0$ is A) 5 B) $\sqrt{5}$ C) $\sqrt{6}$ D) $5\sqrt{5}$ E) -5

SOLUTION

$r = (t^3+1)i + t^2 j + (2t+5)k$

$\frac{dr}{dt} = 3t^2 i + 2tj + 2k$

$\frac{d^2r}{dt^2} = 6ti + 2j + 0k$

at $t = 0$

accel $= 0i + 2j + 0k$

magnitude $= \sqrt{0^2+2^2+0^2}$

$= \sqrt{4} = 2$

No 22: If $r = t^2 i - tj + (2t+1)k$. find $\left|\frac{dr}{dt}\right|_{t=1}$

A) 9 B) 3 C) 4 D) 1 E) 2

SOLUTION

$r = t^2 i - tj + (2t+1)k$

$\frac{dr}{dt} = 2ti - j + 2k$

$\left(\frac{dr}{dt}\right)_{at\ t=1} = 2i - j + 2k$

$\left|\frac{dr}{dt}\right|_{at\ t=1} = \sqrt{2^2+(1)^2+2^2} = 3$

No 14: If $a = 4i + mj + k$, $b = -2i + 2j + 3k$ and $a \times b = -11i - 14j + 2k$. Find m

A) 3 B) 2 C) -2 D) 1 E) -3

SOLUTION

$$a \times b = \begin{vmatrix} i & j & k \\ 4 & m & 1 \\ -2 & 2 & 3 \end{vmatrix}$$

$a \times b = (3m-2)i - (12+2)j + (8+2m)k$

$= -11i - 14j + 2k$ Then

if $3m - 2 = -11$ $m = -3$

No 20: The value of m such that the vectors $3i+mj+2k$ and $5i-j-3k$ are orthogonal is A) 21 B) 9 C) 10 D) −9 E) −10

SOLUTION

$\bar{a}\cdot\bar{b}=0 \rightarrow$ orthogonalism

$(3i+mj+2k)\cdot(5i-j-3k)=0$

$15-m-6=0 \qquad m=-9$

2009 No 37: If $\bar{a}=t^2i-tj+(2t-1)k$, $\bar{b}=(2t-3)i+j-tk$. Find the value of $\frac{d}{dt}(\bar{a}\times\bar{b})$ when $t=1$

A) $2i-j+2k$ B) $2i-k$ C) $7j+3k$ D) $2t^3-5t^2-2t$ E) $-7\bar{j}-2\bar{k}$

SOLUTION

$$\bar{a}\times\bar{b}=\begin{vmatrix} i & j & k \\ t^2 & -t & 2t-1 \\ 2t-3 & 1 & -t \end{vmatrix}$$

$$=(t^2-2t+1)i-(-t^3-(2t-1)(2t-3))j$$
$$+(t^2+t(2t-3))k$$

$$\frac{d}{dt}(\bar{a}\times\bar{b})=(2t-2)i+(3t^2-8t+8)j$$
$$+(2t+4t-3)k$$

at $t=1$

$\frac{d}{dt}(\bar{a}\times\bar{b})=0i+3j+3k$ $\boxed{\text{Cross-check}}$

No 40: The unit vector parallel to the resultant of the vector $\bar{a}=2i+4j-5k$, $\bar{b}=i+2j+3k$

A) $\frac{1}{7}(i+3j-2k)$ B) $3i+6j$ C) $\frac{1}{7}(3i+6j-2k)$ D) $7(3i+6j$

SOLUTION

RESULTANT: $=\bar{a}+\bar{b}=$

$(2i+4j-5k)+(i+2j+3k)$

$=3i+6j-2k$

Unit vector $=\dfrac{3i+6j-2k}{\sqrt{3^2+6^2+(-2)^2}}$

$\dfrac{3i+6j-2k}{\sqrt{9+36+4}}=\dfrac{3i+6j-2k}{7}$ (C)

2007/2008 No 17: Differentiate $\frac{d}{dt}(\bar{a}\times\bar{b})$

A) $\bar{a}\cdot\frac{d\bar{b}}{dt}+\bar{b}\cdot\frac{d\bar{a}}{dt}$

B) $\bar{a}\times\frac{d\bar{b}}{dt}+\bar{b}\times\frac{d\bar{a}}{dt}$

C) $\bar{a}\cdot\frac{d\bar{b}}{dt}+\frac{d\bar{a}}{dt}\cdot\bar{b}$

D) none

E) $\frac{db}{dt}\times\bar{a}+\frac{d\bar{a}}{dt}\times b$

SOLUTION

$\frac{d}{dt}(\bar{a}\times\bar{b})=\bar{a}\times\frac{d\bar{b}}{dt}+\frac{d\bar{a}}{dt}\times\bar{b}$ (D)

WHAT AM I SAYING IN ALL THESE? IF GOD IS FOR YOU WHO CAN BE AGAINST YOU

END TO VECTOR ANALYSIS

WHAT NEXT!!!!!!!

BSF 21: Getting wisdom is the most important thing you can do. Whatever else you get, get insight.

WELCOME TO "DYNAMICS"

When object is projected upward, it is against gravity and $a = -g$. From elementary equation of motion

I) $V = u - gt$

II) $h = ut - \frac{1}{2}gt^2$

III) $V^2 = u^2 - 2gh$

IV) h_t = height attained in tth sec e.g. First, fourth, last sec

$$h_t = u - \frac{1}{2}g(2t-1)$$

V) time of ascending or descending $= t = u/g$

VI) Time of Flight $= 2t$

VII) Maximum height $= \frac{u^2}{2g}$

EXAMPLE

A particle is projected upward from the ground level with velocity 30m/sec find (I) maximum height reached by the particle II) the time taken to return to the ground 3) time taken for the particle to attain a height of 40m above the ground ($g = 10m/s^2$)

SOLUTION

I) $H = \frac{u^2}{2g} = \frac{30^2}{2(10)} = 45m$

II) time taken to return to the ground = time to descend $= t = u/g = \frac{30}{10} = 3sec$

III) $H = ut - \frac{1}{2}gt^2$

$40 = 30t - \frac{1}{2}(10)t^2$

$40 = 30t - 5t^2$

divide through by 5

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 4$ or 2

$t = 4sec$ or $2 sec$

EXAMPLE

A) particle is projected vertically upward with a velocity of Um/s and after t sec another particle is projected upward from the same point and with the same initial velocity. ~~Prove~~ that the particle will meet after a lapse of $\frac{u}{g} + \frac{t}{2}$ sec

SOLUTION

Let the particles meet each other after t_1 sec of projection of the first particle.

∴ Height attained by the first particle in t_1 sec = Height attained by the second particle in $(t_1 - t)$ sec

$H = ut - \frac{1}{2}gt^2$ i.e

$$ut_1 - \frac{1}{2}gt_1^2 = u(t_1 - t) - \frac{1}{2}g(t_1 - t)^2$$

$$\cancel{ut_1} - \frac{1}{2}gt_1^2 = \cancel{ut_1} - ut - \frac{1}{2}g(t_1^2 + t^2 - 2t_1t)$$

$$-\frac{1}{2}gt_1^2 = -ut - \frac{1}{2}gt_1^2 - \frac{1}{2}gt^2 + gt_1t$$

$$-ut = \frac{1}{2}gt^2 - gt_1t$$

$$-ut = \frac{1}{2}g(t^2 - 2t_1t)$$

OR

$$gt_1t = \frac{1}{2}gt^2 + ut$$

divide through by gt

$$t_1 = \frac{1}{2}t + \frac{u}{g} \text{ proved}$$

EXAMPLE

A body falls freely from the top of a tower

and during the last seconds of its motion, it fall $\frac{5}{9}$th of the whole distance prove that the height of the tower is 4.41 metre when $g = 9.8 m/sec^2$

SOLUTION

$$h = ut + \frac{1}{2}gt^2$$

$$u = 0$$

$$h = \frac{1}{2}gt^2 \to (1)$$

at last second

$$h = u + \frac{1}{2}g(2t - 1)$$

$u = 0$ at $\frac{5}{9}$ distance

$$\frac{5}{9}h = \frac{1}{2}g(2t - 1) \to (11)$$

$$h = \frac{9}{10}g(2t - 1) \to (111)$$

$$\frac{1}{2}gt^2 = \frac{9}{10}g(2t - 1)$$

$$\frac{1}{2}t^2 = \frac{9}{10}(2t - 1)$$

$$5t^2 = 18t - 9$$

$$5t^2 - 18t + 9 = 0$$

$$5t^2 - 15t - 3t + 9 = 0$$

$$5t(t - 3) - 3(t - 3) = 0$$

$$(5t - 3)(t - 3) = 0$$

$5t = 3$ or $t = 3$

$t = 3/5$ sec or 3 sec

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 3^2$$

$$h = 44.1m$$

PAUSE!!! GIVE THANKS TO GOD

a lapse of $\frac{u}{g} + \frac{t}{2}$ sec

SOLUTION

Let the particles meet each other after t_1 sec of projection of the first particle.

$\therefore$ Height attained by the first particle in t_1 sec = Height attained by the second particle in $(t_1 - t)$ sec

$H = ut - \frac{1}{2}gt^2$ i.e

$ut_1 - \frac{1}{2}gt_1^2 = u(t_1 - t) - \frac{1}{2}g(t_1 - t)^2$

$\cancel{ut_1} - \frac{1}{2}gt_1^2 = \cancel{ut_1} - ut - \frac{1}{2}g(t_1^2 + t^2 - 2t_1t)$

$-\frac{1}{2}gt_1^2 = -ut - \frac{1}{2}gt_1^2 - \frac{1}{2}gt^2 + gt_1t$

$-ut = \frac{1}{2}gt^2 - gt_1t$

$-ut = \frac{1}{2}g(t^2 - 2t_1t)$

OR

$gt_1t = \frac{1}{2}gt^2 + ut$

divide through by gt

$t_1 = \frac{1}{2}t + \frac{u}{g}$ proved

EXAMPLE

A body falls freely from the top of a tower and during the last seconds of its motion, it fall $\frac{5}{9}$th of the whole distance prove that the height of the tower is 4.41 metre when $g = 9.8 m/sec^2$

SOLUTION

$h = ut + \frac{1}{2}gt^2$

$u = 0$

$h = \frac{1}{2}gt^2 \rightarrow (1)$

at last second

$h = u + \frac{1}{2}g(2t-1)$

$u = 0$ at $\frac{5}{9}$ distance

$\frac{5}{9}h = \frac{1}{2}g(2t-1) \rightarrow (ii)$

$h = \frac{9}{10}g(2t-1) \rightarrow (iii)$

$\frac{1}{2}gt^2 = \frac{9}{10}g(2t-1)$

$\frac{1}{2}t^2 = \frac{9}{10}(2t-1)$

$5t^2 = 18t - 9$

$5t^2 - 18t + 9 = 0$

$5t^2 - 15t - 3t + 9 = 0$

$5t(t-3) - 3(t-3) = 0$

$(5t-3)(t-3) = 0$

$5t = 3$ or $t = 3$

$t = 3/5$ sec or 3 sec

$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 3^2$

$h = 44.1m$

PAUSE !!! GIVE THANKS TO GOD

$t = u/g = \frac{49}{9.8} = 5sec$

Answer is B

2007/2008 : No 32 : What is the time taken by a body falling from rest through a vertical distance of 490m A) 10sec B) 5sec C) 15sec D) 25sec E) 20sec

SOLUTION

$H = ut + \frac{1}{2}gt^2$

$u = 0$

$490 = \frac{1}{2} \, 9.8 \, t^2$

$t^2 = \frac{490}{9.8} \times 2$

$t^2 = 50 \times 2$

$t^2 = 100$

$t = 10sec$

Answer is A

27: If a particle moves in the north east direction with a velocity of 6cm/sec and has an acceleration of 8cm/s² toward the north 6cm/s² toward ~~the~~ east. Find the distance travelled by the particle after the lapse of 2sec A) 23cm B) 54cm C) 30cm D) 32cm E) 45.

SOLUTION

By pythagoras

$a = g = 10$

$t = 2sec$

$V = u + gt \quad u = 6cm/s$

$V = 6 + gt = 6 + 20 \rightarrow (1)$

$V = 26$

$H = ut + \frac{1}{2}at^2$

$H = 6 \times 2 + \frac{1}{2} \times 10 \times 2^2$

$H = 12 + 20 = 32cm$

Answer is D

No 25: For a particle to attain the greatest height h when projected vertically upward the least velocity of projection (initial) velocity must be A) $\frac{\sqrt{8h}}{2}$ B) $\frac{8h}{2}$ C) $2\sqrt{gh}$ D) $\frac{h}{2g}$ E) $\sqrt{2gh}$

SOLUTION

$V^2 = u^2 - 2gh$

$V = 0$

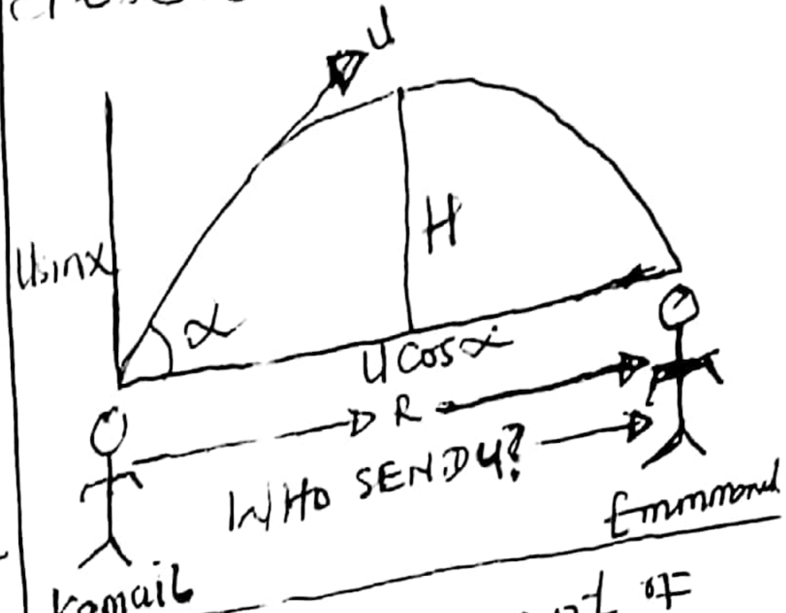
$u^2 = 2gh \quad u = \sqrt{2gh}$

Answer is E

PVT 23: The more you talk, the more likely you are to sin. If you are wise, you will keep quiet

PROJECTILE

Imagine, we were plucking mango and my friend threw a big stone at the mango. It landed on my shoulder in the other side of the tree. The path taken by the stone is parabolic (parabola curve) in nature. The highest point reached by the stone before it started descending is called **Maximum Height (H)**. The horizontal distance from my friend shoulder to my shoulder (Assuming we are of the same height) is called **Range (R)**. The total time taken for the stone to ascend to the maximum height and to descend to my shoulder is **Time of Flight (T)**. Time taken to ascend to the maximum height equal to the time taken to descend to my shoulder. Time of flight is twice time taken to ascend or descend (t)

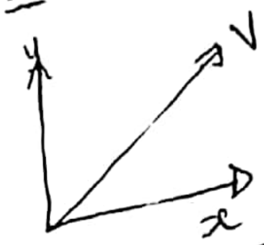


Horizontal component of the velocity $= U\cos\alpha \rightarrow$ (I)

Vertical component of the velocity $= U\sin\alpha \rightarrow$ (II)

$x = u\cos\alpha \rightarrow$ (III)

$y = u\sin\alpha - gt \rightarrow$ (IV)



x and y are velocity component resolved

$$V = \sqrt{x^2 + y^2}$$

$$V = \sqrt{(U\cos\alpha)^2 + (u\sin\alpha - gt)^2}$$

$$V = \sqrt{U^2\cos^2\alpha + U^2\sin^2\alpha + g^2t^2 - 2u\sin\alpha gt}$$

$$V = \sqrt{U^2(\cos^2\alpha + \sin^2\alpha) + g^2t^2 - 2u\sin\alpha gt}$$

$$V = \sqrt{U^2 - 2u\sin\alpha gt + g^2t^2}$$

$V = \sqrt{u^2 - 2g(ut\sin\alpha - \frac{1}{2}gt^2)}$

$\tan\theta = \frac{y}{x} = \frac{u\sin\alpha - gt}{u\cos\alpha}$

Velocity at height t

$V = \sqrt{u^2 - 2g((u\sin\alpha)t - \frac{1}{2}gt^2)}$

where

$h = u\sin\alpha t - \frac{1}{2}gt^2$

$V = \sqrt{u^2 - 2gh}$

DISPLACEMENT

$\frac{dx}{dt} = u\cos\alpha$

$\int dx = \int u\cos\alpha \, dt$

$x = ut\cos\alpha + C$

at $t = 0$ $C = 0$

$x = (u\cos\alpha)t \rightarrow (I)$

$\frac{dy}{dt} = u\sin\alpha - gt$

$\int dy = \int (u\sin\alpha - gt)\, dt$

$y = (u\sin\alpha)t - \frac{1}{2}gt^2 \rightarrow (II)$

y and x are displacement resolved

y = vertical displacement

x = horizontal displacement

from eq (I)

$t = \frac{x}{u\cos\alpha} \rightarrow (III)$

SO

Substitute eq (III) in (II)

$y = (u\sin\alpha)t - \frac{1}{2}gt^2$

$y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$

$y = (\tan\alpha)x - \frac{g}{2u^2}(\sec^2\alpha)x^2$

Compared with

$y = ax + bx^2$ [eqn of parabola]

at $a = 0$

$y = bx^2$ [such as $x^2 = 4ay$]

or $x^2 = \frac{1}{b}y$

SUMMARY

$t = \frac{u\sin\alpha}{g} \rightarrow (I)$

$T = \frac{2u\sin\alpha}{2g} \rightarrow (II)$

$R = \frac{u^2\sin 2\alpha}{g}$

R is maximum at (I) $\alpha = 45°$ (II) $\sin 2\alpha = 1$ (III) $\tan\alpha = 1$

$H = \frac{u^2\sin^2\alpha}{2g}$

H is maximum at $\alpha = 90°$

$\alpha = \pi/2$ or $\sin\alpha = 1$

t = time taken to ascend or descend (return)

T = time of flight

R = Range

H = maximum height

Scanned by CamScanner

BSF14: Whenever you possibly can, do good to those who need it. You dont know may be its because of it God is still keeping you here on earth.

SOLVED PAST QUESTION

2001/2002 No 49. A particle is projected with a velocity $100ms^{-1}$ at an angle of $30°$ to the horizontal. Show that the greatest height attained by the particle is 125m ($g = 10m/s^2$)

SOLUTION

$$H = \frac{U^2 \sin^2 \alpha}{2g} = \frac{100^2 (\sin 30)^2}{2 \times 10}$$

$$H = \frac{100^2 \times (0.5)^2}{20} = 125m$$

2007/2008 No34: If R is the horizontal range and T is the time of flight of a projectile, then the angle of projection $\tan\alpha$ equals A) $\frac{gT^2}{R}$ B) $\frac{gT^2}{2R}$ C) $\frac{T^2}{Rg}$ D) $\frac{T^2}{2Rg}$ E) gT^2R.

$$R = \frac{U^2 \sin 2\alpha}{g}$$

$$R = \frac{U^2 (2\sin\alpha\cos\alpha)}{g} \rightarrow (I)$$

$$H = \frac{U^2 \sin^2\alpha}{2g} \rightarrow (II)$$

$$T = \frac{2U\sin\alpha}{g} \rightarrow (III)$$

square both side of eq (III)

$$T^2 = \frac{4U^2(\sin\alpha)(\sin\alpha)}{g^2} \rightarrow (IV)$$

divide eq (I) by (IV)

$$\frac{R}{T^2} = \frac{\frac{\cancel{U^2}(\cancel{2}\cancel{\sin\alpha}\cos\alpha)}{g}}{\frac{\cancel{4}\cancel{U^2}(\cancel{\sin\alpha})(\sin\alpha)}{g^2}}$$

$$\frac{R}{T^2} = \frac{g\cos\alpha}{2\sin\alpha}$$

Find the inverse of both side

$$\frac{2\sin\alpha}{g\cos\alpha} = \frac{T^2}{R}$$

$$\frac{2}{g}\tan\alpha = \frac{T^2}{R}$$

$$\tan\alpha = \frac{gT^2}{2R}$$

answer is B

No 35: A particle is projected with a velocity of 40ft/sec in a direction making angle 30° degree with the horizontal. Find its velocity after 0.5 sec (g = 32ft/sec)
A) 32ft/sec B) 30ft/sec
C) $8\sqrt{19}$ ft/sec D) $16\sqrt{5}$ ft/sec
E) $8\sqrt{9}$ ft/sec

SOLUTION

$$V = \sqrt{u^2 - 2u\sin\alpha g t + g^2t^2}$$

$u = 40 \quad \alpha = 30 \quad t = 0.5$

$$V = \sqrt{40^2 - 2(40)\sin 30(32)(0.5) + (32)^2(0.5)^2}$$

$$V = \sqrt{1600 - 640 + 256}$$

$$V = \sqrt{1216}$$

$$V = \sqrt{64 \times 19} = 8\sqrt{19}$$

answer is C

2009: No 3: If the horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Find the angle of projection A) 60° B) 45° C) 15° D) 90° E) 30°

SOLUTION

$$R = 4\sqrt{3}\,H$$

$$\frac{\cancel{u^2}\sin 2\alpha}{\cancel{g}} = 4\sqrt{3}\,\frac{\cancel{u^2}\sin^2\alpha}{\cancel{2g}}$$

$$\frac{\cancel{2}\sin\alpha\cos\alpha}{\cancel{g}} = \cancel{2}\sqrt{3}\,\frac{\cancel{\sin\alpha}\sin\alpha}{\cancel{g}}$$

$$\cos\alpha = \sqrt{3}\sin\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{1}{\sqrt{3}}$$

$$\tan\alpha = 1/\sqrt{3}$$

$$\alpha = \tan^{-1} 1/\sqrt{3} = 30°$$

answer is E

No 29: A body is projected with a velocity of 80m/sec at an angle 30° with the horizontal, find the time of flight $g = 10m/s^2$
A) 16sec B) 8sec C) 4sec D) 12sec
E) 6sec

SOLUTION

$$T = \frac{2u\sin\alpha}{g}$$

$$= \frac{2 \times 80 \sin 30}{10} = 8sec$$

Answer is B

...s; Yes, beg for knowledge, plead for insight. Look for as hard as you would look for silver or hidden treasure.

No 30: A body is projected with velocity of 40m/sec at an angle 30° with the horizontal. Find the horizontal range. $g = 10m/s^2$ A) $80\sqrt{3}$m B) $1280\sqrt{3}$m C) $640\sqrt{3}$m D) $320\sqrt{3}$m E) $160\sqrt{3}$m

SOLUTION

$$R = \frac{u^2 \sin 2\alpha}{g}$$

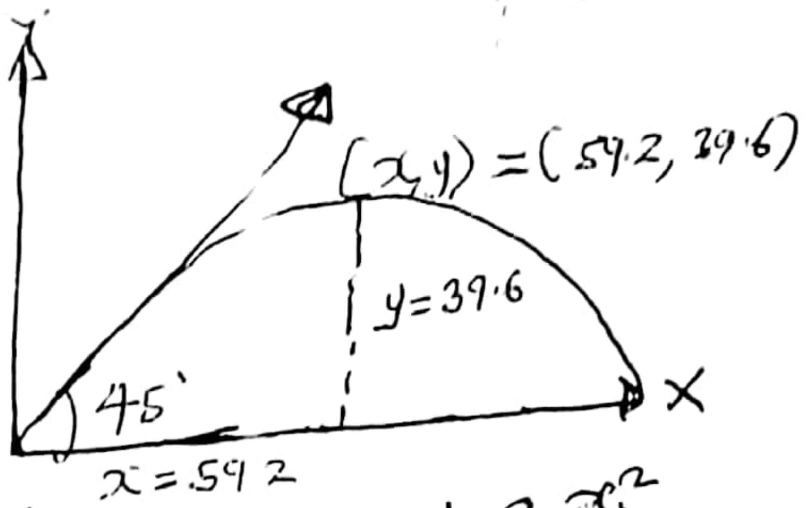
$$R = \frac{40^2 \sin 2(30)}{10}$$

$$= \frac{160 \sin 60}{10} = 80\sqrt{3}\,m$$

Answer is A

No 39: A village boy aims at a bird sitting at the top of a tree 39.6m high and 59.2m off. If the direction of projectile make an angle of 45°. Find the velocity with which the arrow hits the bird ($g = 9.8m/s^2$) A) 2.96m B) $2.96\sqrt{2}$m C) $29.6\sqrt{2}$m D) $\frac{2.96}{\sqrt{2}}$m E) none

SOLUTION



$$y = x\tan\alpha - \frac{1}{2}\frac{g x^2}{u^2\cos^2\alpha}$$

$$y = x\tan 45 - \frac{1}{2}\frac{g x^2}{(\cos 45)^2 u^2}$$

$$y = x - \frac{1}{\cancel{2}}\frac{g x^2}{u^2} \times \cancel{2}$$

$$y = x - \frac{1}{u^2} g x^2$$

$y = 39.6$ $x = 59.2$

$$39.6 = 59.2 - \frac{1}{u^2} \times 9.8\,(59.2)^2$$

$$\frac{9.8 \times (59.2)^2}{u^2} = 59.2 - 39.6$$

$$u^2 = \frac{9.8 \times (59.2)^2}{19.6}$$

$$u^2 = \frac{(59.2)^2}{2}$$

$$u = \sqrt{\frac{(59.2)^2}{2}} = \frac{59.2}{\sqrt{2}}$$

$$u = \frac{59.2\sqrt{2}}{2} = 29.6\sqrt{2}$$

$u = 29.6\sqrt{2}$ m/s

Answer is C

PAUSE!!! GIVE THANKS TO GOD

2010: A bullet is projected with a velocity of 98m/s at angle of 30° with the horizontal if $g = 9.8m/s^2$

No 29: The horizontal range is A) $490\sqrt{3}m$ B) $490\sqrt{2}m$ C) $49\sqrt{3}m$ D) $480\sqrt{3}m$ E) $4.9\sqrt{3}m$

SOLUTION

$$R = \frac{U^2 \sin 2\alpha}{g}$$

$$R = \frac{98^2 \sin 2(30)}{9.8}$$

$\sin 60 = \sqrt{3}/2$

$$R = \frac{98^2 \sin 60}{9.8} = 490\sqrt{3}m$$

Answer is A

No 30: The greatest height attained is A) 1225m B) 1235m C) 122.5m D) 123.5m E) 12.35m

SOLUTION

$$H = \frac{U^2 \sin^2\alpha}{2g}$$

$$H = \frac{98^2 (\sin 30)^2}{2(9.8)}$$

$$H = 122.5m$$

Answer is C

No 31: The time of fly is A) 5sec B) 10sec C) 15. D) 20sec E) 17sec

SOLUTION

$$T = \frac{2u \sin\alpha}{g}$$

$$T = \frac{98 \sin 30}{9.8}$$

$$T = 5sec$$

Answer is A

No 32: The velocity at height of 100m is A) $\sqrt{7044}$ m/sec B) $\sqrt{7444}$ m/sec C) $\sqrt{7644}$ m/sec D) $\sqrt{7046}$ m/sec E) none

SOLUTION

$$H = \frac{U^2 \sin^2\alpha}{2g}$$

$$100 = \frac{U^2 (\sin 30)^2}{2(9.8)}$$

$$1960 = U^2 \times \frac{1}{4}$$

$$U^2 = 4 \times 1960$$

$$U^2 = 7840$$

$$u = \sqrt{7840} \text{ m/sec}$$

Answer is E

If you fail to learn what ought to know, you will learn what you ought not to know

BSF 26: Being Cheerful keeps you healthy. It is slow death to be gloomy all the time. Cheer Up!!!

MOMENTUM

Impulse $= Ft = m(V-U)$

Impulse $\Rightarrow$ change in momentum

TYPES OF COLLISION

ELASTIC COLLISION: both energy and momentum are conserved. Coefficient of Restitution for perfectly elastic collision $e = 1$

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2 \rightarrow (i)$$

$$-e = \frac{V_2 - V_1}{U_2 - U_1} \rightarrow (ii)$$

$$e = 1$$

$$-1 = \frac{V_2 - V_1}{U_2 - U_1}$$

$$\therefore V_2 + U_2 = V_1 + U_1 \rightarrow (iii)$$

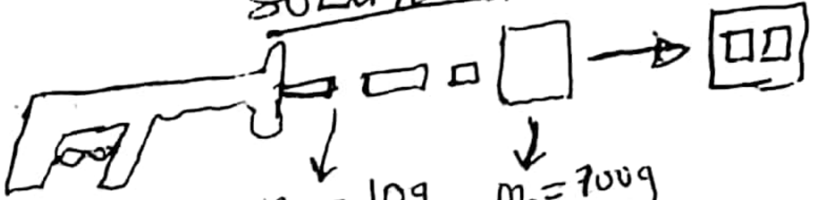
INELASTIC COLLISION: Energy is lost while momentum is conserved and $e < 1$. Example is when objects moves with common velocity after collision

$$m_1U_1 + m_2U_2 = (m_1 + m_2)V$$

EXAMPLE

A bullet of mass 10grammes is fired with a horizontal velocity of 330m/s into a block of wood of mass 100grammes which is free to move on a smooth horizontal table. If the bullet is buried in the wood and both wood and bullet move with common velocity V m/s. Find the value of V

SOLUTION



$m_1 = 10g$ $m_2 = 100g$

$U_1 = 330$ $U_2 = 0$

$$m_1U_1 + m_2U_2 = (m_1 + m_2)V$$

$$0.01kg(330) + 0.1kg(0) = (0.01 + 0.1)V$$

$$0.11V = 3.3$$

$$V = \frac{3.3}{0.11} = 30m/s$$

EXAMPLE

A mass of 3kg travelling at 5m/s is acted on by a force of 10N for 0.3sec in direction of motion. What is the final velocity of the body

SOLUTION

$Ft = m(V_2 - V_1)$

$10(0.3) = 3(V_2 - 5)$

$3 = 3V_2 - 15$

$3V_2 = 18 \quad V_2 = 6m/s$

EXAMPLE

A body whose mass is 4.5Kg is acted upon by a force which changes its velocity from 36Km/hr to 54Km/hr. Find the impulse of the Force

SOLUTION

$V_1 = 36Km/hr = \frac{36 \times 1000}{3600} = 10m/s$

$V_2 = 54Km/hr = \frac{54 \times 1000}{3600} = 15m/s$

$Impulse = m(V_2 - V_1)$

$= 4.5(15-10) = 22.5Ns$

EXAMPLE

A ball of mass 0.5kg is thrown toward a wall so that it strikes the wall normally with a speed of 10m/s. If the ball bounces at right angle away from the wall with a speed of 8m/s. What impulse does the wall exert on the ball

SOLUTION

bounced back → opposite direction

$V_2 = -8m/s \quad V_1 = 10m/s$

$Impulse = m(V_2 - V_1)$

$= 0.5(-8-10) = -9Ns$

EXAMPLE

A ball of mass 8kg and moving with velocity of 4m/s overtakes a ball of mass 10kg moving with velocity of 2m/s in the same direction. If the coefficient of restitution $e = 1/2$ find the velocity of the balls after impact

SOLUTION

Assuming they are moving in the opposite direction both U_2 and V_1 are negative

$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$

$8(4) + 10(2) = 8V_1 + 10V_2$

$8V_1 + 10V_2 = 52 \rightarrow (i)$

$-e = \frac{V_2 - V_1}{U_2 - U_1}$

$-1/2 = \frac{V_2 - V_1}{2-4}$

$V_2 - V_1 = 1 \rightarrow (ii)$

$V_2 = 1 + V_1$

$8V_1 + 10(1+V_1) = 52$

$18V_1 = 52 - 10$

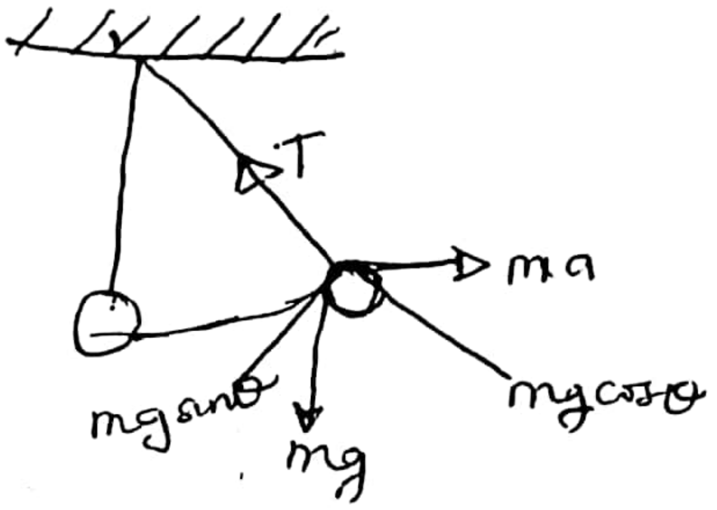
$V_1 = \frac{42}{18} = 7/3 m/s$

$V_2 = 1 + V_1 = 1 + 7/3$

$V_2 = 10/3 m/s$

BSF: Finally my dear friend, whatever you find yourself doing, do it very well to the Glory of God

SIMPLE PENDULUM



$$T = mg\cos\theta \longrightarrow (1)$$

• accel $= \frac{d^2s}{dt^2}$

$$F = m \times a = m \times \frac{d^2s}{dt^2}$$

$$\cancel{m}\frac{d^2s}{dt^2} = -\cancel{m}g\sin\theta$$

[-ve means opposite direction]

$$\boxed{\frac{d^2s}{dt^2} = -g\sin\theta}$$ — saw

θ is very small

$\sin\theta \longrightarrow \theta$

$$\therefore \frac{d^2s}{dt^2} = -g\theta$$

$$\theta = s/r = \frac{s}{L}$$

$$\frac{d^2s}{dt^2} = -g\frac{s}{L}$$

g/L is constant $= \mu$

File → $$\boxed{\frac{d^2s}{dt^2} = -\mu s}$$

File shows equation of S.H.M (simple harmonic motion)

Its period $T = \frac{2\pi}{\sqrt{\mu}}$

$$T = \frac{2\pi}{\sqrt{g/L}}$$

Chisel → $$\boxed{T = 2\pi\sqrt{L/g}}$$

HOOKE'S LAW

$$\boxed{T_e = \lambda\left(\frac{x}{L}\right)}$$ — metre rule

T_e = tension in a string

x = extension

L = original length

λ = modulus of elasticity

cutlass

$$\boxed{W = \frac{1}{2}(T_{e_1}+T_{e_2})x}$$

EXAMPLE

An elastic string of natural length 2m and modulus of elasticity 6N is stretched until the extending force of magnitude 4N. How much has been done and at what is the final extension

SOLUTION

$$T_e = \lambda\frac{x}{L}$$

$$x = T_e\frac{L}{\lambda} = \frac{4\times 2}{6}$$

$$x = 4/3$$

$$W = \frac{1}{2}(T_{e_1}+T_{e_2})x$$

$$T_{e_1} = 0$$

$$W = \frac{1}{2}\times 4\times\frac{4}{3} = \frac{8}{3}\,J$$

EXAMPLE

An elastic string of natural length 2m is fixed at one end is stretched to 2.8m in length by a force of 4N. What is the modulus of elasticity

SOLUTION

$$T = \frac{\lambda(x-L)}{L}$$

$$4 = \frac{\lambda(2.8-2)}{2}$$

$$\lambda = \frac{4\times 2}{0.8} = 10N$$

SOLVED PAST QUESTION

2007/2008 No 36: An elastic string of natural length 5m is fixed at one end and is stretched to 5.5m in length by a force of 2N. Find the modulus of elasticity A) 2N B) 30N C) 10N D) 20N E) 25N

$$T = \frac{\lambda(x-L)}{L}$$

$$\lambda = \frac{TL}{x-L} = \frac{2(5)}{5.5-5}$$

$$\lambda = 20N$$

Answer is D

No 32 (2009). The motion of simple pendulum is governed by the equation.

A) $\frac{d^2s}{dt^2} = g\cos\theta$ B) $\frac{d^2s}{dt^2} = g\sin\theta$

C) $\frac{d^2s}{dt^2} = -g\sin\theta$ D) $\frac{d^2s}{dt^2} = -g\cos\theta$

E) $\frac{ds}{dt} = g\sin\theta$

SOLUTION

Bring out your sau

Answer is C

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